Name______________________________ Date____________________ Period______

Worksheet 4.5—Exponential and Log Equations

Show all work on a separate sheet of paper. All answers must be given as either simplified, exact answers. No calculator is permitted unless otherwise stated.

Multiple Choice

1. Solve $2^{3x-1}=32$

   (A) $x=1$ (B) $x=2$ (C) $x=4$ (D) $x=11$ (E) $x=13$

2. Solve $\ln x=-1$

   (A) $x=-1$ (B) $x=\frac{1}{e}$ (C) $x=1$ (D) $x=e$ (E) DNE

3. The domain of the function $f(x)=\log_4(5x+3)-2$ over the set of real numbers is

   (A) $(-1.4,\infty)$ (B) $(-0.6,\infty)$ (C) $(-\infty,\infty)$ (D) $(-2.6,\infty)$ (E) $\left(-1\frac{2}{3},\infty\right)$

4. If $B=\ln\left(\frac{1}{1-x}\right)+\ln\left(\frac{1}{1+x}\right)$, then $e^B=$

   (A) $\ln\left(\frac{1}{1-x^2}\right)$ (B) $\frac{2}{1-x^2}$ (C) $\frac{x}{1-x^2}$ (D) $\ln\left(\frac{x}{1-x^2}\right)$ (E) $\frac{1}{1-x^2}$

5. If $\log_3(x+3)+\log_3 x=\log_3 28$, then x equals

   (A) 3 (B) 4 (C) 7 (D) 12 (E) 21

6. If $3^{x+y}=9$ and $3^{x-y}=9$ then xy equals

   (A) 6 (B) 0 (C) 5 (D) 7.2 (E) 1

7. Evaluate $(\log_2 4)(\log_4 8)(\log_8 16)$

   (A) 2 (B) 4 (C) 8 (D) (E) 32

8. If $(\log_a b)(\log_b 3)(\log_3 d)(\log_d 0.125)=-1.5$, then the value of a is

   (A) -5 (B) 5 (C) 4 (D) -4 (E) 0.5

9. If $\frac{1}{25}$ of 5^{20} is 125^x, then the value of x is

   (A) -3 (B) 5^3 (C) -5 (D) 6 (E) 2^3

10. If $5^{x+y}=6$ and $5^{x-y}=4$, then $25^x=?$

    (A) $2\sqrt{6}$ (B) 10 (C) 20 (D) 24 (E) 125

All work is on next page.

1) $\log_2 32 = 3x-1$

$5 = 3x-1$

$3x=6$

$x=2$

2) D: $x>0$

$e^{-1} = x$

$x = \frac{1}{e}$

3) D: x

$5x+3>0$

$5x>-3$

$x>-\frac{3}{5}$

4) $x>$

$\ln\left(\frac{1}{(1-x)(1+x)}\right) = B$

$e^B = \frac{1}{(1-x)(1+x)}$

$= \frac{1}{1-x^2}$

5) $\log_3(x^2+3x) = \log_3 28$

Domain

$x^2+3x>0$

$x(x+3)$

$x>0$

$x>-3$

$x^2+3x=28$

$x^2+3x-28=0$

$(x+7)(x-4)=0$

$\cancel{x=-7}$ $x=4$

6) $3^{x+y} = 3^2$

$3^{x-y} = 3^2$

$\begin{cases} x+y=2 \\ x-y=2 \end{cases}$

$2x=4$

$x=2$ $y=0$

$2\cdot 0=0$

7) $\frac{\log 4}{\log 2} \cdot \frac{\cancel{\log 8}}{\cancel{\log 4}} \cdot \frac{\log 16}{\cancel{\log 8}}$ $\frac{\log 16}{\log 2} = 4$

8) $\frac{\cancel{\log b}}{\log a} \cdot \frac{\cancel{\log 3}}{\cancel{\log b}} \cdot \frac{\cancel{\log d}}{\cancel{\log 3}} \cdot \frac{\log 0.125}{\cancel{\log d}} = -1.5$

$\frac{\log 0.125}{\log a} = -1.5$

$\log_a 0.125 = -1.5$

$(a^{-1.5})^{-\frac{2}{3}} = (0.125)^{-\frac{2}{3}}$

$a=4$

9) $\frac{1}{25} \cdot 5^{20} = 125^x$

$5^{-2} \cdot 5^{20} = 5^{3x}$

$5^{18} = 5^{3x}$

$3x=18$

$x=6$

10) $5^{x+y} = 6$ $5^{x-y} = 4$

$\log_5 5^{x+y} = \log_5 6$ $\log_5 5^{x-y} = \log_5 4$

$x+y = \log_5 6$ $x-y = \log_5 4$

$\begin{cases} x+y = \log_5 6 \\ x-y = \log_5 4 \end{cases}$

$2x = \log_5 6 + \log_5 4$

$\frac{2x}{2} = \frac{\log_5(24)}{2}$

$x = .987$ (sto→x)

$25^x = 24$

or

$5^{x+y} \cdot 5^{x-y} = 6\cdot 4$

$5^{2x} = 24$

$(5^2)^x = 24$

$25^x = 24$

e) $x > 0, x > 2$

$\log_2(3x) = \log_2(5x-10)$

$3x = 5x - 10$

$-2x = -10$

$\boxed{x = 5}$

g) D: $x > 5$, $x > -3$

$\log_9[(x-5)(x+3)] = 1$

$9^1 = x^2 - 2x - 15$

$0 = x^2 - 2x - 24$

$0 = (x-6)(x+4)$

$x = 6$, $\cancel{-4}$

f) $5^{\log_5\left(\frac{x+1}{x-1}\right)} = 5^2$

D: $x > -1$, $x > 1$

$x-1\left(\frac{x+1}{x-1} = 25\right)$

$x + 1 = 25x - 25$

$26 = 24x$

$\boxed{x \neq \frac{12}{13}}$

no solution

12. Using a **calculator**, solve each of the following to 3 decimals:

(a) $\log_9 x = x^2 - 2$ (b) $e^{x^2} - 2 > x^3 - x$

a) $x = 0.0123, 1.475$ b. $\{x \mid x > .885 \text{ or } x < 0.706\}$

13. Evaluate each of the following:

(a) $\log_{49} 7 - \log_8 64$ (b) $\log_3 \sqrt{243\sqrt{81\sqrt[3]{3}}}$

a) $\frac{1}{2} - 2 = \frac{-3}{2}$ b) $\log_3 3^{\frac{5}{2}} \cdot 3^{\frac{4}{4}} \cdot 3^{\frac{1}{12}}$

$\log_3 3^{\frac{43}{12}} = \frac{43}{12}$

Short Answer:

11. For each of the following, find the simplified, exact solution accompanied by a three-decimal approximation (if applicable).

(a) $2\cdot 3^{x/4} = 5\cdot 7^{(1-x)}$ (b) $\frac{10}{1-e^{-x}} = 2$ (c) $4x^3e^{3x} = 3x^4e^{3x}$ (d) $e^x - 12e^{-x} - 1 = 0$

(e) $\log_2 3 + \log_2 x = \log_2 5 + \log_2 (x-2)$ (f) $\log_5 (x+1) - \log_5 (x-1) = 2$

(g) $\log_9 (x-5) + \log_9 (x+3) = 1$

a) $\ln(2\cdot 3^{\frac{x}{4}}) = \ln(5\cdot 7^{(1-x)})$

$\ln 2 + \frac{x}{4}\ln 3 = \ln 5 + (1-x)\ln 7$

$\ln 2 + \frac{x}{4}\ln 3 = \ln 5 + \ln 7 - x\ln 7$

$\frac{x}{4}\ln 3 + x\ln 7 = -\ln 2 + \ln 5 + \ln 7$

$\frac{x(\frac{1}{4}\ln 3 + \ln 7)}{(\frac{1}{4}\ln 3 + \ln 7)} = \frac{-\ln 2 + \ln 5 + \ln 7}{(\frac{1}{4}\ln 3 + \ln 7)}$

$x = \frac{-\ln 2 + \ln 5 + \ln 7}{4^{-1}\ln 3 + \ln 7}$

c) $4x^3e^{3x} = 3x^4e^{3x}$

$4x^3e^{3x} - 3x^4e^{3x} = 0$

$x^3e^{3x}(4-3x) = 0$

$x^3 = 0$ $e^{3x} = 0$ $4-3x=0$

$x=0$ never $x = \frac{4}{3}$

b) $\frac{10}{1-e^{-x}} = 2$

$10 = 2(1-e^{-x})$

$10 = 2 - 2e^{-x}$

$8 = -2e^{-x}$

$-4 = e^{-x}$

$\ln 4$

no solution

D) $e^x - 12e^{-x} - 1 = 0$

$e^x - 12e^{-x} = 1$

$\left(e^x - \frac{12}{e^x} = 1\right)e^x$

$e^{2x} - 12 = e^x$

$e^{2x} - e^x = 12$

$e^{2x} - e^x - 12 = 0$

$(e^x - 4)(e^x + 3) = 0$

$\ln e^x = \ln 4$ $e^x = -3$ not possible

$x = \ln 4$

14. Solve for x:

(a) $2\log_b x = 2\log_b(1-a) + 2\log_b(1+a) - \log_b\left(\frac{1}{a}-a\right)^2$

(b) $\log_b x = 2 - a + \log_b\left(\frac{a^2 b^a}{b^2}\right)$ (c) $\log_2(x-4) + \log_{\sqrt{2}}(x^3-2) + \log_{1/2}(x-4) = 20$

(d) $\log_5(\ln(x+3)-1) + \log_{1/5}(\ln(x+3)-1) = 0$

a) Domain $1-a>0$ $\quad a<1$ $\quad 1+a>0$ $\quad a>-1$ $\quad \left(\frac{1}{a}-a>0\right)^a$ $\quad 1-a^2>0$

$D: \{a \mid -1<a<1\}$

$\log_b x^2 = \log_b\left(\frac{(1-a)^2(1+a)^2}{\left(\frac{1}{a}-a\right)^2}\right)$

$(x^2)^{\frac{1}{2}} = \left(\frac{(1-a)^2(1+a)^2}{\left(\frac{1}{a}-a\right)^2}\right)^{\frac{1}{2}}$ $\quad x = \frac{(1-a)(1+a)}{\frac{1}{a}-a}\left(\frac{a}{a}\right) = \frac{a\cancel{(1-a^2)}}{\cancel{(1-a^2)}} = a$

b) $\log_b x = 2 - a + \log_b(a^2 b^a b^{-2})$

$\log_b x = 2 - a + \log_b a^2 + \log_b b^a + \log_b b^{-2}$

$\log_b x = \cancel{2} \cancel{-a} + 2\log_b a \cancel{+a} \cancel{-2}$

$\log_b x = \log_b a^2$

$\boxed{x = a^2}$

c) $\log_2(x-4) + \frac{\log_2(x^3-2)}{\log_2 2^{\frac{1}{2}}} + \frac{\log_2(x-4)}{\log_2 2^{-1}} = 20$

$\log_2(x-4) + \frac{\log_2(x^3-2)}{\frac{1}{2}} + \frac{\log_2(x-4)}{-1} = 20$

$\log_2(x-4) + 2\log_2(x^3-2) - \log_2(x-4) = 20$

$\log_2\left(\frac{(x-4)(x^3-2)^2}{(x-4)}\right) = 20$

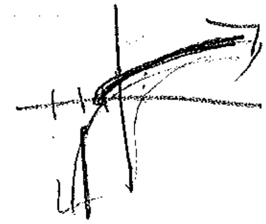
$\log_2(x^3-2)^2 = 20$

$(2^{20})^{\frac{1}{2}} = ((x^3-2)^2)^{\frac{1}{2}}$

$2^{10} = x^3 - 2$

$x^3 = 2^{10} + 2$ $\quad x = (2^{10}+2)^{\frac{1}{3}}$

d) $\log_5(\ln(x+3)-1) + \dfrac{\log_5(\ln(x+3)-1)}{\log_5 5^{-1}} = 0$



$\underbrace{\log_5(\ln(x+3)-1)} - \underbrace{\log_5(\ln(x+3)-1)} = 0$

like terms

$0 = 0$

Domain $(\ln(x+3)-1) > 0$

$\ln(x+3)-1 = 0$

$\ln(x+3) = 1$

$e^1 = x+3$

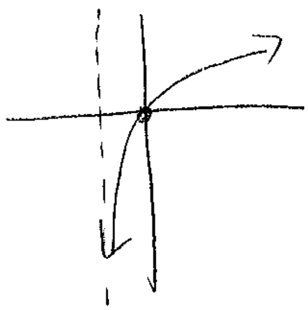
$x = e-3$

$\therefore$ True for all values in the Domain

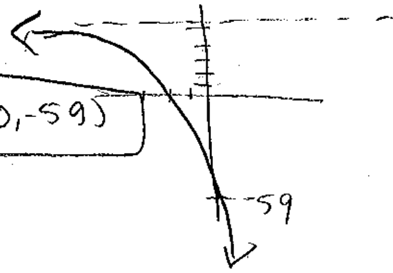
$\{x \mid x > e-3\}$

15. Find the x- and y-intercepts of the following, then sketch the graphs.
(a) $y = 2\log_3(x+1)$ (b) $y = 5 - 4^{x+3}$

a) $y = 2\log_3(1)$
$y = 2(0)$
$y = 0$
$(0,0)$



b) $y = 5 - 4^{0+3}$
$y = 5 - 4^3$
$y = -59$ $\boxed{(0,-59)}$



$0 = 5 - 4^{x+3}$
$-5 = -4^{x+3}$
$\log_4 5 = \log_4 4^{x+3}$
$\log_4 5 = x+3$
$x = \log_4 5 - 3$
$(\log_4 5 - 3, 0)$

16. Find the inverse of the following functions.
(a) $g(x) = 5 + \log_3(2x+2)$ (b) $f(x) = e^{x+2} - 1$

$x = 5 + \log_3(2y+2)$
$x - 5 = \log_3(2y+2)$
$3^{x-5} = 2y+2$
$\frac{3^{x-5} - 2}{2} = \frac{2y}{2}$
$y = \frac{3^{x-5} - 2}{2}$
$y =$

$x = e^{y+2} - 1$
$\ln(x+1) = \ln e^{y+2}$
$\ln(x+1) = y+2$
$y = \ln(x+1) - 2$

17. Solve the following **literal equations** for the indicated variable.

(a) $T = T_s + D_0 e^{-kt}$ for k (b) $y = \dfrac{a}{1+be^{-(x-c)/d}}$ for d (c) $y = ae^{-(x-b)^2/c}$ for c

a)

$$T = T_S + D_0 e^{-kt}$$

$$\frac{T - T_S}{D_0} = \frac{D_0 e^{-kt}}{D_0}$$

$$\ln \frac{T-T_S}{D_0} = \ln e^{-kt}$$

$$\frac{\ln\left(\frac{T-T_S}{D_0}\right)}{-k} = \frac{-kt}{\cancel{t}}$$

$$\boxed{-\frac{1}{t}\ln\left(\frac{T-T_S}{D_0}\right) = K}$$

or

$$K = \ln\left(\frac{D_0}{T-T_S}\right)^{\frac{1}{t}}$$

b)

$$y = \frac{a}{1+be^{-(x-c)/d}}$$

$$\frac{\left(1+be^{-(x-c)/d}\right)y}{y} = \frac{a}{y}$$

$$1+be^{-(x-c)/d} = \frac{a}{y}$$

$$\frac{be^{-(x-c)/d}}{b} = \frac{\frac{a}{y}-1}{b}$$

$$\ln e^{-(x-c)/d} = \ln \frac{\frac{a}{y}-1}{b}$$

$$(d)-(x-c)/d = \ln\left(\frac{\frac{a}{y}-1}{b}\right)(d)$$

$$\frac{-(x-c)}{\ln\left(\frac{\frac{a}{y}-1}{b}\right)} = \frac{\ln\left(\frac{\frac{a}{y}-1}{b}\right)d}{\ln\left(\frac{\frac{a}{y}-1}{b}\right)}$$

$$d = \frac{-x-c}{\ln\left(\frac{\frac{a}{y}-1}{b}\right)}$$