Name______________________________ Date____________________ Period______

Worksheet 4.2—Exponential and Logistic Modeling

Show all work on a separate sheet of paper. All answers must be given as either simplified, exact answers or approximations with 3-decimal accuracy. Calculators ARE permitted.

Multiple Choice

1. What is the percent growth rate of $M(t) = 1.25 \cdot 1.049^t$

   (A) 49% (B) 23% (C) 4.9% (D) 2.3% (E) 1.23%

2. What is the percent decay rate of $q(k) = 22.9 \cdot 0.834^k$

   (A) 22.7% (B) 16.6% (C) 8.34% (D) 2.27% (E) 0.834%

3. A single-cell amoeba doubles every 4 days. About how long will it take one amoeba to produce a population of 1000?

   (A) 10 days (B) 20 days (C) 30 days (D) 40 days (E) 50 days

   $f(t) = 1 \cdot (2)^{\frac{x}{4}}$

   $1000 = 2^{\frac{x}{4}}$

   $(4)\frac{\ln 1000}{\ln 2} = \frac{x}{4}(4)$

4. The number of children infected with typhoid in a small village is modeled by the logistic equation $R(t) = \dfrac{789}{1+16e^{-0.8t}}$, R is the number of children infected after t days. Based on this model, which of the following is true?

   (A) After 0 days, 16 children are infected (B) After 2 days, 439 children are infected
   (C) After 4 days, 590 children are infected (D) After 6 days, 612 children are infected
   (E) After 8 days, 769 children are infected

5. Which exponential function models decay with an initial value of 12, decreasing at a rate of 0.47% per week?

   (A) $S(t) = 47(0.0012)^t$ (B) $S(t) = 12(0.0047)^t$ (C) $S(t) = 12(0.9953)^t$
   (D) $S(t) = 47(0.0995)^t$ (E) $S(t) = (0.47)^t$

6. Which exponential function models decay with an initial value of 0.7 g, doubling every 3 days.

   (A) $S(t) = 0.7(2)^t$ (B) $S(t) = 0.7(2)^{t/3}$ (C) $S(t) = 0.7(3)^{t/7}$
   (D) $S(t) = 0.7(7)^t$ (E) $S(t) = 0.7(2)^{3t}$

   $S(t) = .7(2)^{\frac{x}{3}}$

   $S(t) = 4b^2$

7. A quantity Q grows exponentially over time t. At time $t = 2$, $Q = 16$ grams, and time $t = 5$, $Q = 128$ grams. How much is Q at $t = 3$? $\boxed{Q = 32}$

   $16 = A(b)^2$

   $128 = A(b)^5$

   (A) 6 grams (B) 16 grams (C) 10 grams (D) 2 grams (E) 8 grams

8. A substance grows exponentially as $N(t) = Ne^{rt}$, where $N(t)$ is the quantity of the substance after t hours and N is the original quantity of the substance. If the substance grows from 700 grams to 2100 grams in 3 hours, find the weight/mass of the substance after 9 hours. Triples every 3 days

   (A) 18903 grams (B) 18900 grams (C0 18927 grams (D) 700 grams (E) 700.632 grams

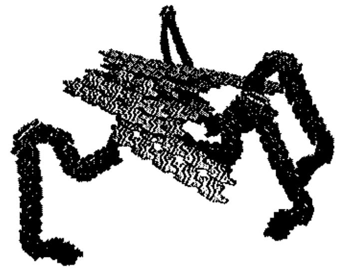
   (0, 700)
   (3, 2100)

   $\boxed{n(t) = 700(3)^{\frac{x}{3}}}$

Short Answer

$R(t) = 6(4)^{\frac{x}{3}}$

9. A evil cloning replicator reproduces itself at a rate that the population of replicators quadruples every 3 hours. At $t = 0$, there are 6 evil cloning replicators.
   (a) Write an equation for the number of replicators $R(t)$ at time t hours.
   (b) How many replicators are there after 48 hours?
   (c) After how many hours will the number of replicators reach 1,000,000? How many days is this?



a) $R(t) = 6(4)^{\frac{x}{3}}$

b) $R(48) = 2.577 \times 10^{10}$

c) $\frac{6(4)^{\frac{x}{3}}}{6} = \frac{1{,}000{,}000}{6}$

$$\frac{\frac{x}{3}\ln 4}{\ln 4} = \frac{\ln\left(\frac{1{,}000{,}000}{6}\right)}{\ln 4}$$

$$x = \frac{\ln\left(\frac{1{,}000{,}000}{6}\right)(3)}{\ln 4}$$

$x \approx 26.02$ hours ≈ 1.084 days

10. The half-life of a radioactive isotope describes the amount of time that it takes half of the isotope in a sample to decay. In the case of radiocarbon dating, the half-life of carbon 14 is 5,730 years. A fossil is found that has 35% carbon 14 compared to the living sample. How old is the fossil?

$A(t) = 1(.5)^{\frac{x}{5{,}730}}$

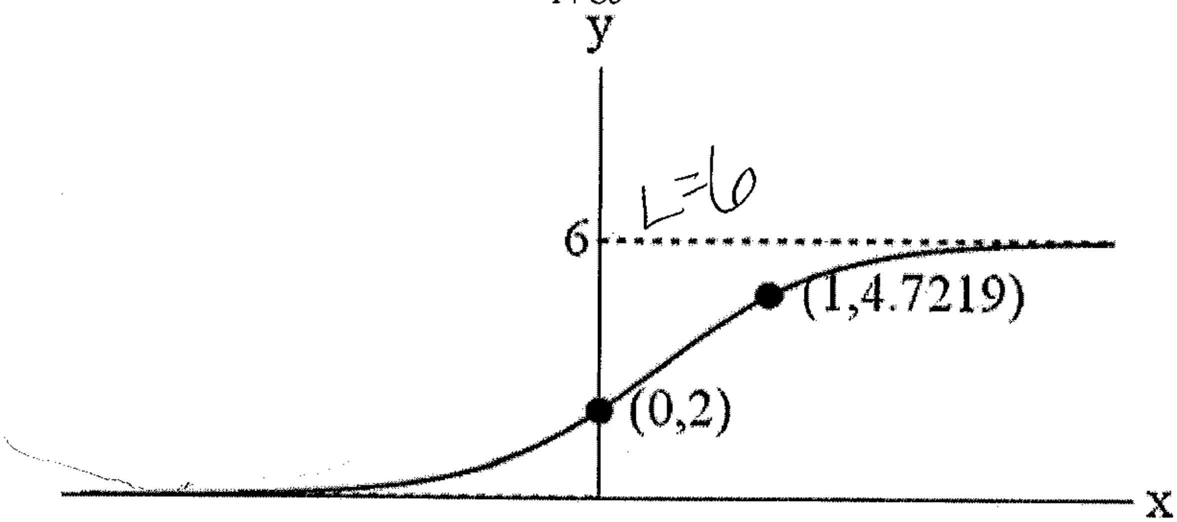
$.35 = .5^{\frac{x}{5730}}$

$$\frac{\ln .35}{\ln .5} = \frac{x}{5730}$$

$$5730\left(\frac{\ln .35}{\ln .5}\right) = x$$

$x = 8{,}678.504$ years

11. Determine an equation of the form $f(x) = \frac{L}{1+Ce^{-kx}}$ for the function whose graph is shown below.



$$2 = \frac{L}{1+Ce^{-k(0)}} \Rightarrow (1+C)\,2 = \frac{6}{1+C}\,(1+C) \Rightarrow 2+2C=6$$

$$2C=4$$

$$\underline{C=2}$$

$$4.7219 = \frac{L}{1+Ce^{-k(1)}} \Rightarrow 4.7219 = \frac{6}{1+2e^{-k(1)}}$$

$$\frac{(1+2e^{-k(1)})\,4.7219}{4.7219} = \frac{6}{4.7219}$$

$$1+2e^{-k} = \frac{6}{4.7219}-1$$
$$-1$$

$$\ln e^{-k} = \ln \frac{\frac{6}{4.7219}-1}{2}$$

$$-k=-2$$

$$\underline{k=2}$$

$$\boxed{f(x) = \frac{6}{1+2e^{-2x}}}$$