Name Key Date______________ Period______

Worksheet 3.5—Rational Functions

Show all work on a separate sheet of paper. All answers must be given as **simplified, exact answers**! No Calculators are permitted unless specified otherwise.

Multiple Choice

1. Let $f(x) = -\frac{2x}{x^2+3x}$. For what values of x does the graph of $f(x)$ have a vertical asymptote? $= \frac{-2x}{x(x+3)}$ $x+3=0$ $x=-3$

   (A) $x=0$ (B) $x=0$, $x=3$ (C) $x=3$ (D) $x=-3$ (E) $x=0$, $x=-3$

2. Let $f(x) = -\frac{2x^2}{x^2+3x-4}$. Which of the following is an equation of an asymptote of $f(x)$? $= \frac{2x^2}{(x+4)(x-1)}$ $x+4=0$ $x-1=0$ $x=-4$ $x=1$

   (A) $y=2$ (B) $x=1$ (C) $x=4$ (D) $x=-2$ (E) $y=-4$

3. Let $f(x) = \frac{x^2}{x+5}$. Which of the following statements is true about the graph of f?

   (A) There is ~~no~~ VA (B) There is an HA but no VA (C) There is an SA but no VA (D) There is a VA and an SA (E) There is a VA and an HA

4. What is the degree of the end-behavior model of $f(x) = \frac{x^8+1}{x^4+1}$?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

5. The equation of the end-behavior model of $f(x) = \frac{2x^3-x+6}{x+2}$ is given by

   (A) $y=2x^2-7$ (B) $y=2x^2-1$ (C) $y=2x^2+4x+7$ (D) $y=2x^2-4x+7$ (E) $y=2x^2-4x-7$

$$\begin{array}{r|rrrr} -2 & 2 & 0 & -1 & 6 \\ & & -4 & 8 & -14 \\ \hline & 2 & -4 & 7 & \boxed{-8} \end{array}$$

$2x^2-4x+7$

Short Answer

6. Find the x- and y- intercepts of the following functions

   (a) $t(x) = \frac{x^2-x-2}{x-6}$ (b) $r(x) = \frac{x^3-9x}{x^3}$

a) x-int

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2$ $x=-1$

y int

$t(0) = \frac{(0)^2-(0)-2}{(0)-6}$

$= \frac{-2}{-6}$

$= \frac{1}{3}$

xint: $(2,0)(1,0)$ yint $(0,\frac{1}{3})$

b. xint

$x(x^2-9)=0$

$x(x+3)(x-3)=0$

$x = \cancel{0}, -3, 3$ $x \neq 0$

yint

$r(0) = \frac{(0)^3-9(0)}{(0)^3}$

$=$ none

Xint: $(-3,0)(3,0)$

7. Find all vertical and horizontal asymptotes (if any).

(a) $k(x)=\dfrac{6x-2}{x^2+5x-6}$ (b) $j(x)=\dfrac{3x^2}{5+2x+x^2}$ (c) $careful(x)=\dfrac{2x+x^3}{x-1}$

a) $=\dfrac{2(3x-1)}{(x+6)(x-1)}$

VA! $x+6=0$ $x-1=0$

$x=-6$ $x=1$

HA! $y=0$

b) $=\dfrac{3x^2}{x^2+2x+5}$

VA: None

HA $y=3$

c) $=\dfrac{x(x^2+2)}{x-1}$

VA: $x-1=0$

$x=1$

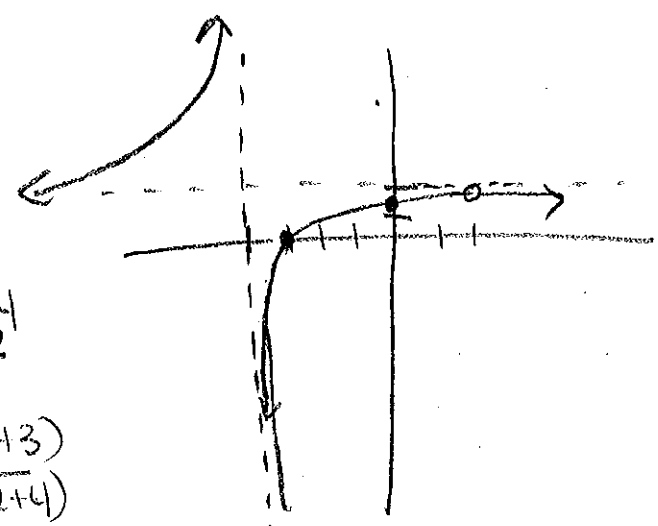
HA: none

8. Analyze the following functions. As in the notes, find the domain, discontinuities, intercepts, and end-behavior. Sketch a graph. Find the equations of all HA's, VA's, and SA's. Give the coordinate of any hole. Find the range after you graph it.

(a) $f(x)=\dfrac{4x^2+4x-24}{2x^2+4x-16}$

$f(x)=\dfrac{4(x^2+x-6)}{2(x^2+2x-8)}$

$=\dfrac{4(x+3)\cancel{(x-2)}}{2(x+4)\cancel{(x-2)}}$



Df $\{x \mid x \neq -4, 2\}$

VA @ $x=-4$

HA @ $y=2$

hole 2

$=\dfrac{4(2+3)}{2(2+4)}$

$=\dfrac{10}{6}$

$\boxed{\text{hole@}\left(2, \tfrac{5}{3}\right)}$

Rf $\{y \mid y \neq 2\}$

$x+3=0$

$x=-3$

$\boxed{\text{xint: }(-3,0)}$

$y=\dfrac{-24}{-16}$

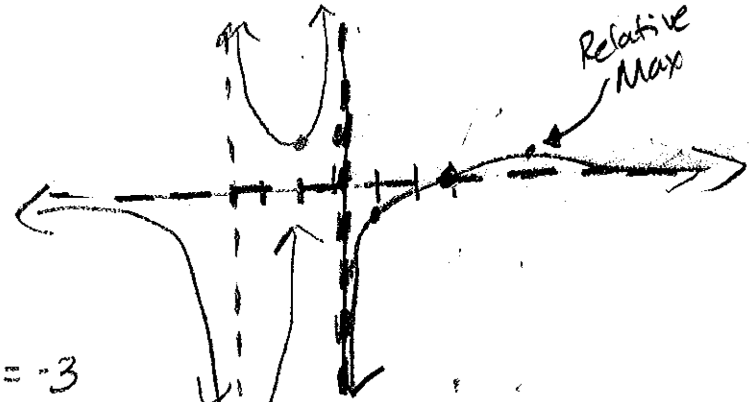
$y=\frac{3}{2}$

$\boxed{\text{yint: }\left(0,\tfrac{3}{2}\right)}$

$\lim_{x\to\infty} f(x)=2$

$\lim_{x\to-\infty} f(x)=2$

(b) $h(x) = \frac{x-3}{x^2+3x}$



$h(x) = \frac{x-3}{x(x+3)}$

Dh: $\{x \mid x \neq 0, -3\}$ VA @ $x = 0$, $x = -3$

xint: $(3, 0)$ HA @ $y = 0$

yint: $x \neq 0$ so no yint.

$\lim_{x \to \infty} h(x) = 0$ (on pos side)

$\lim_{x \to -\infty} h(x) = 0$ (on neg side

$f(-1) = \frac{-4}{-2} = 2$ To see what's happening here

$f(1) = \frac{-2}{4} = -\frac{1}{2}$ To see what's happening her

Range? Can't find. Don't know the Relative Maximum

(c) $q(x)=\frac{2x^3-6x^2-14x}{x^2+3x}$

$q(x)=\frac{2x(x^2-3x-7)}{x(x+3)}$

$D_q: \{x \mid x \neq -3, 0\}$

VA @ $x=-3$

hole $=\frac{2(0^2-3(0)-7)}{(0+3)}$

$=\frac{-14}{3}$

hole @ $(0, -\frac{14}{3})$

y int: none $x \neq 0$

x int: $x^2-3x-7=0$

$x=\frac{3 \pm \sqrt{(-3)^2-4(1)(-7)}}{2(1)}$

x int $=\frac{3 \pm \sqrt{37}}{2}$

For graph x int. are about $\frac{3 \pm 6}{2}$ so $\frac{9}{2}$ + $\frac{-3}{2}$

SA:

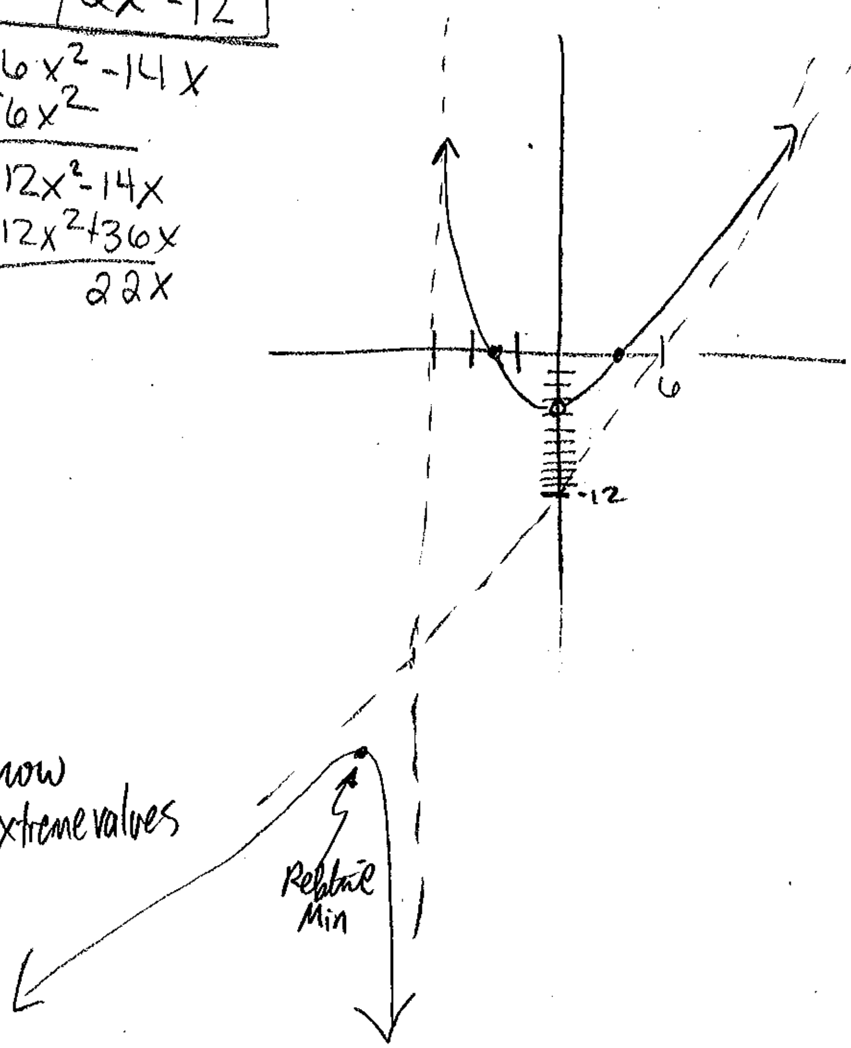
$$\begin{array}{r} \boxed{2x-12} \\ x^2+3x \,\overline{)\, 2x^3-6x^2-14x} \\ -2x^3+6x^2 \\ \hline -12x^2-14x \\ +12x^2+36x \\ \hline 22x \end{array}$$

SA @ $y=2x-12$

$\lim_{x \to \infty} q(x)=\infty$

$\lim_{x \to -\infty} q(x)=-\infty$

Range? Don't know Relative Extreme values



(d) $t(x)=\frac{(x^2-x-2)(x-3)}{x^2-4x+3}$

$t(x)= \frac{(x-2)(x+1)(x-3)}{(x-3)(x-1)}$

$D_t: \{x \mid x \neq 3, 1\}$

VA @ $x=1$

hole

$= \frac{(3-2)(3+1)}{(3-1)}$

$= \frac{4}{2}$

hole @ $(3,2)$

y int $t(0)=\frac{((0)^2-(0)-2)(0-3)}{(0)^2-4(0)+3}$

$= \frac{6}{3}$

$= 2$

y int: $(0,2)$

x int $\quad x-2=0 \quad x+1=0$

$x=2 \quad x=-1$

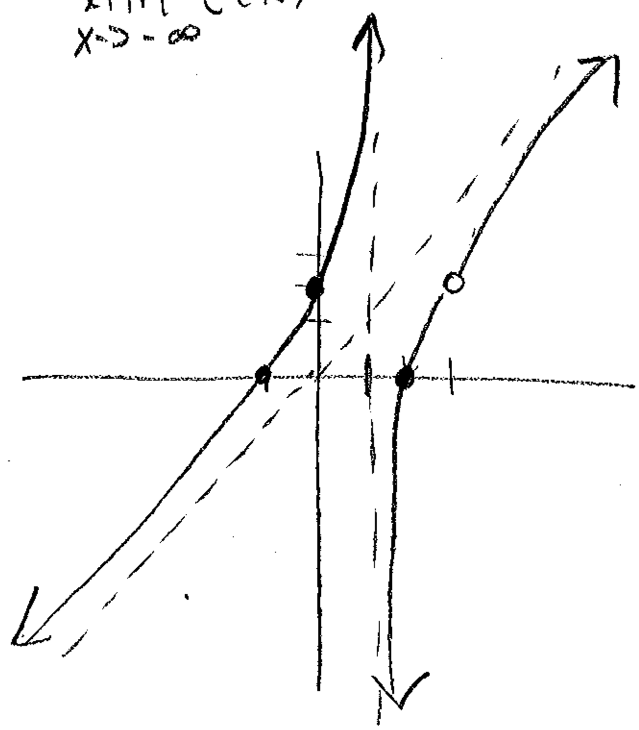
SA:

$$\begin{array}{r|l} & \boxed{x} \\ x-1 & x^2-x-2 \\ & -x^2+x \\ \hline & -2 \end{array}$$

SA: $y=x$

$\lim_{x\to\infty} t(x) = \infty$

$\lim_{x\to-\infty} t(x) = -\infty$



Range: $\mathbb{R}$

9. Write and equation of a function, $f(x)$, with a VA at $x=-1$, a hole at $x=3$, and x-intercept at $x=-3$, and an HA at $y=1$. Once you have the equation, find $\lim_{x\to 3} f(x)$.

$$\boxed{f(x)=\frac{(x+3)(x-3)}{(x+1)(x-3)}}$$

To find $\lim_{x\to 3} f(x)$

$$=\frac{3+3}{3+1}$$

$$=\frac{6}{4}$$

$$\boxed{\lim_{x\to 3} f(x) = \frac{3}{2}}$$

10. Write an equation of a function $d(x)$ with a y-intercept of $(0,-2)$, a VA at $x=1$, an SA at $y=2x+7$, and a hole at $x=2$, As $x\to\infty$, what do the slopes of the graph of $d(x)$ approach?

$$d(x)=\frac{(2x^2+5x+c)(x-2)}{(x-1)\ (x-2)}$$

So $d(x)=\dfrac{f(x)(x-2)}{(x-1)(x-2)}$ and $\dfrac{f(x)}{x-1}=2x+7+\dfrac{R(x)}{x-1}$ $R(x)\neq 0$

So $f(x)=(2x+7)(x-1)+R(x)$, $R(x)\neq 0$

$f(x)=2x^2+5x-7+R(x)\neq 0$

So $\underbrace{f(x)=2x^2+5x+C}$, $C\neq -7$

*As $x\to\infty$ the slopes of $d(x)$ approach 2 because the slope of the S.A. is $m=2$.

To find C: $d(0)=-2=\dfrac{C}{-1}$ so $\boxed{C=2}$

$$\text{So } \boxed{d(x)=\frac{(2x^2+5x+2)(x-2)}{(x-1)\ (x-2)}}$$

11. Analyze and sketch $h(x) = \frac{x^5 - 1}{x + 2}$. Show all asymptotes, including end-behavior asymptotes.

VA $X = -2$

SA:

-2	1	0	0	0	0	-1
		-2	4	-8	16	-32
	1	-2	4	-8	16	-33

end behavior $X^4 - 2x^3 + 4x^2 - 8x + 16$

$\lim_{X \to \infty} h(x) = \infty$

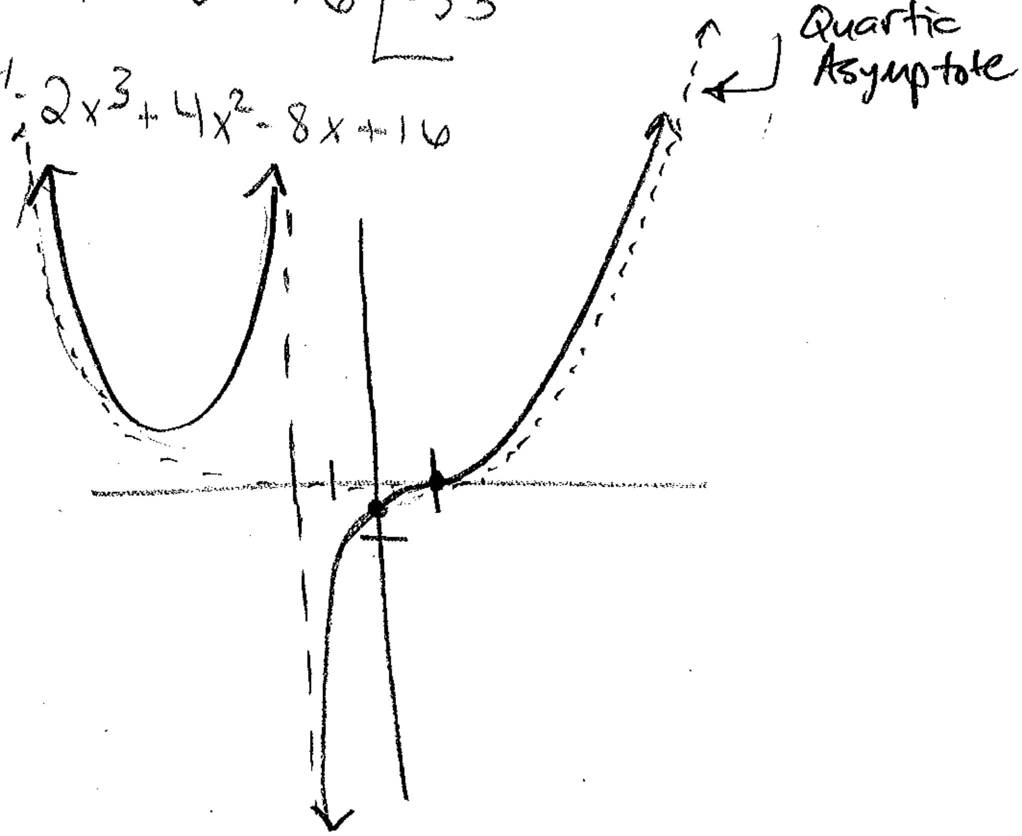
$\lim_{X \to -\infty} h(X) = -\infty$

X int $X^5 - 1 = 0$

$X^5 = 1$

$X = 1$

y int $(0, -\frac{1}{2})$



12. (Calculator permitted) A drug is administered to a patient, and the concentration of the drug in the bloodstream is monitored. At time $t \geq 0$ (in hours since giving the drug), the concentration (in mg/L) is given by

$$c(t) = \frac{5t}{t^2 + 1}$$

Graph the function with your graphing calculator in a reasonable window.

(a) What is a reasonable X and Y window? Justify.

(b) What is the highest concentration of drug that is reaching in the patient's bloodstream? How do you know this?

(c) What happens to the drug concentration after a long period of time? What are the mathematical implications of this if the person lives for many, many, many years after the injection?

(d) What is the concentration after 5 hours?

(e) How long does it take for the concentration to drop below 0.3 mg/L?

a) X(t) is time in hours $0 \leq t \leq 48$ (2 days?)
y(t) is concentration $0 \leq c \leq 3$ 0 is lowest amt 2.5 is max

b) relative max is 2.5 mg/L (max on calc)

c) The concentration approaches 0 after a long period of time. They will always have a trace of the drug in their bloodstream

d) .961 mg/L = c(5)

e) $\boxed{t = 16.606 \text{ hours}}$; since $c(t) < 0.3$ for $t > 16.606$ hours