Name________________ Date__________ Period____

Worksheet 3.4—Complex Zeros of Polynomial Functions

Show all work on a separate sheet of paper. All answers must be given as **simplified, exact answers**! Calculators are permitted, but only to help you narrow down choices of rational zeros.

Multiple Choice

1. The reciprocal of the number i is
   (A) $-i$ (B) -1 (C) 1 (D) i (E) none of these

   $\frac{1}{i}\cdot\frac{i}{i} = \frac{i}{i^2} = -i$

2. State the **possible** number of imaginary zeros of $g(x) = x^5 + 3x^3 + 7x^2 - 6x - 13$
   (A) 3 or 1 (B) 2, 4, or 0 (C) Exactly 1 (D) Exactly 3 (E) Exactly 4

3. What is the simplified form of $\frac{9i^5 - 5i^{1735}}{2i^4}$
   (A) $-7i$ (B) $-2i$ (C) $2i$ (D) $7i$ (E) none of these

   $\frac{9i}{2} - \frac{5i^{1735}}{2}$ $\frac{9i - 5(-i)}{2}$ $\frac{14i}{2} = 7i$

   $1735 \div 4 = 433$, remainder 3

4. $\frac{(-2-2i)(5+2i)}{(5-2i)(5+2i)} =$ (A) $-\frac{2}{5}$ (B) $-\frac{14}{29}-\frac{14}{29}i$ (C) $-\frac{6}{29}-\frac{14}{29}i$ (D) $-\frac{6}{21}-\frac{14}{21}i$ (E) $-\frac{14}{21}-\frac{14}{21}i$

   $-10i$

   $\frac{-10-14i+4}{25+4} = \frac{-6-14i}{29}$

Short Answer

#3 was changed.

If you have $5i^{1735}$ then the answer is E

If you have the newer version then the answer is D.

4. List all possible rational x-intercepts of $y = 2x^3 + 3x - 5$, then find all complex roots. Use your calculator to narrow down your rational root possibilities. Show the synthetic division.

$\frac{5}{2}$ $\quad \frac{\pm 1, \pm 5}{\pm 1, \pm 2}$, $\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}$ 8 possible

$$\begin{array}{r|rrrr} 1 & 2 & 0 & 3 & -5 \\ & & 2 & 2 & 5 \\ \hline & 2 & 2 & 5 & \boxed{0} \end{array}$$

$2x^2 + 2x + 5$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(5)}}{4}$$

$$x = \frac{-2 \pm \sqrt{-36}}{4} = \quad x = \frac{-2 \pm 6i}{4} \quad x = -\frac{1}{2} \pm \frac{3}{2}i$$

$\boxed{\text{Roots: } x = 1, -\frac{1}{2} \pm \frac{3i}{2}}$

5. Use the Rational Roots Test to find possible rational zeroes of $y = 6x^4 - 11x^3 + 8x^2 - 33x - 30$, then find the all complex roots. Use your calculator to narrow down your rational root possibilities. Show the synthetic division. You do not have to list the rational root possibilities.

$y = 6x^4 - 11x^3 + 8x^2 - 33x - 30$ $\quad \boxed{\text{Roots: } x = -\frac{2}{3}, \frac{5}{2}, \pm\sqrt{3}\,i}$

$$\begin{array}{r|rrrrr} -\frac{2}{3} & 6 & -11 & 8 & -33 & -30 \\ & & -4 & 10 & -12 & 30 \\ \hline \frac{5}{2} & 6 & -15 & 18 & -45 & \boxed{0} \\ & & 15 & 0 & 45 & \\ \hline & 6 & 0 & 18 & \boxed{0} & \end{array}$$

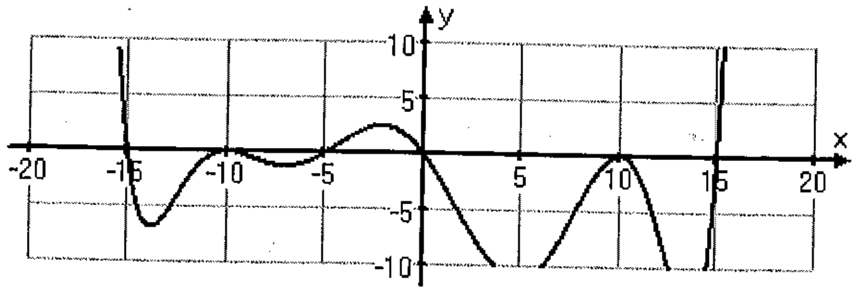
$6x^2 + 18 = 0$

$6(x^2 + 3) = 0$

$$\frac{-0 \pm \sqrt{0 - 4(1)(3)}}{2} = \frac{\pm\sqrt{-12}}{2} \;{}^{-4 \cdot 3} = \pm\sqrt{3}\,i$$

calculator permitted to find A

6. The following is a graph of a 8^{th} degree polynomial function, $f(x)$, with all real roots.
   (a) Write a general equation of the function.



   (b) If the function satisfies $f(-2)=3$, find the particular equation of $f(x)$. Show work and use proper notation.

$$f(x) = A\,x(x+15)(x+10)^2(x+5)^3(x-10)^2(x-15)$$

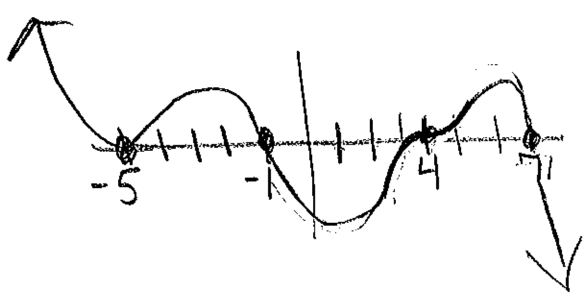
$$3 = A(-2)(13)(8)^2(3)^3(-12)^2(-17)$$

$$3 = A(10983744)$$

$$\boxed{f(x) = \frac{1}{3661248}x(x+15)(x+10)^2(x+5)^3(x-10)^2(x-15)}$$

7. Sketch a graph of the following polynomial. Show all x-intercepts.

$$y = -{}^{1}/_{5600}(x+5)^2(x+1)(x-4)^3(x-7)$$



8. Given that $-i+2$ is a zero of $f(x)=x^5-6x^4+11x^3-x^2-14x+5$, find all complex roots using synthetic division. List your possible rational roots also.

Possible $\pm 1, \pm 5$

$$\begin{array}{r|rrrrrr} 2-i & 1 & -6 & 11 & -1 & -14 & 5 \\ & & 2-i & -9+2i & 6+2i & 12-i & -5 \\ \hline 2+i & 1 & -4-i & 2+2i & 5+2i & -2-i & \boxed{0} \\ & & 2+i & -4-2i & -4-2i & 2+i & \\ \hline & 1 & -2 & -2 & 1 & \boxed{0} & \end{array}$$

$x^3-2x^2-2x+1=0$

Possible roots: ± 1

Hoping for 1 :)

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -2 & 1 \\ & & 1 & -1 & -3 \\ \hline & 1 & -1 & -3 & \boxed{-2} \end{array}$$

:(

so must be -1

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -2 & 1 \\ & & -1 & 3 & -1 \\ \hline & 1 & -3 & 1 & \boxed{:)} \end{array}$$

Ha!

$x^2-3x+1=0$

$$x=\frac{3\pm\sqrt{9-4}}{2}$$

$$x=\frac{3\pm\sqrt{5}}{2}=\frac{3}{2}\pm\frac{\sqrt{5}}{2}$$

$$\boxed{\text{Roots: } x=2\pm i,\ -1,\ \frac{3}{2}\pm\frac{\sqrt{5}}{2}}$$

9. Find all the complex zeroes of the following polynomial: $f(x) = 2x^5 + 3x^4 - 30x^3 - 57x^2 - 2x + 24$. List all possible rational roots first, then use your calculator to help narrow down the search. Show your synthetic division.

factors of $\frac{24}{2}$ $\quad \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24}{\pm 1, \pm 2}$

Possible Roots: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24, \pm \frac{1}{2}, \pm \frac{3}{2}$

Found from graph $4, -\frac{3}{2}, -1$

$$
\begin{array}{r|rrrrrr}
4 & 2 & 3 & -30 & -57 & -2 & 24 \\
 & & 8 & 44 & 56 & -4 & -24 \\
\hline
-1 & 2 & 11 & 14 & -1 & -6 & \boxed{0} \\
 & & -2 & -9 & -5 & 6 & \\
\hline
-\frac{3}{2} & 2 & 9 & 5 & -6 & \boxed{0} & \\
 & & -3 & -9 & 6 & & \\
\hline
 & 2 & 6 & -4 & \boxed{0} & &
\end{array}
$$

$2x^2 + 6x - 4 = 0$

$2(x^2 + 3x - 2) = 0$

$x = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 \pm \sqrt{17}}{2}$

$$\boxed{\text{Roots: } x = 4, -1, -\frac{3}{2}, \frac{-3}{2} \pm \frac{\sqrt{17}}{2}}$$

10. Find the remainder when $x^{36}+4x^{27}+7$ is divided by $x+1$.

$(-1)^{36}+4(-1)^{27}+7$

$1-4+7=3$

remainder is 3

11. Find $P(x)$ if $P(x)$ divided by $x-1$ has a remainder of -2 and a quotient of x^3+x^2-x-1. Write $P(x)$ in expanded form.

$$\boxed{P(x)=(x^3+x^2-x-1)+\frac{-2}{x-1}}$$

12. A polynomial function has the following complex roots: A polynomial $P(x)$ has the following roots: $-2,\ 1+\sqrt{3};\ 5i$.

(a) Write an equation of the function of lowest possible degree. Remember to expand any factors containing radicals or imaginary units.

(b) If $P(0)=-25$, write the particular equation.

$P(x)=A(x+2)(x-1-\sqrt{3})(x-1+\sqrt{3})(x-5i)(x+5i)$

$x^2-x+\cancel{x\sqrt{3}}-x+1-\cancel{\sqrt{3}}-\cancel{x\sqrt{3}}+\cancel{\sqrt{3}}-3$

$P(x)=A(x+2)(x^2-2x-2)(x^2+25)$

$-25=A(2)(-2)(25)$

$\frac{-25}{-100}=\frac{A(-100)}{-100}$

$A=\frac{1}{4}$

$$\boxed{P(x)=\frac{1}{4}(x+2)(x-2x-2)(x^2+25)}$$

13. Determine a polynomial of lowest degree with real coefficients that has the given roots:

(a) $0(m2),\ 4+3i$ (b) $7-i\sqrt{5}\ (m2),\ \sqrt{5}\ (m2)$

(a)

$f(x) = Ax^2(x-4-3i)(x-4+3i)$

$x^2 - 4x + 3ix - 4x + 16 - 12i - 3ix + 12i + 9$

$f(x) = Ax^2(x^2 - 8x + 25)$

(b) $f(x) = A\left((x-7+i\sqrt{5})(x-7-i\sqrt{5})\right)^2\left((x-\sqrt{5})(x+\sqrt{5})\right)^2$

$x^2 - 7x - \cancel{ix\sqrt{5}} - 7x + 49 + \cancel{7i\sqrt{5}} + \cancel{ix\sqrt{5}} - \cancel{7i\sqrt{5}} + 5$

$\boxed{f(x) = A(x^2 - 14x + 54)^2(x^2 - 5)^2}$

14. Determine k so that $f(x) = x^3 - 11x^2 + kx - 6$ has $x-3$ as a factor.

$0 = (3)^3 - 11(3)^2 + k(3) - 6$

$0 = 27 - 99 + 3k - 6$

$0 = -78 + 3k$

$78 = 3k$

$\boxed{k = 26}$

15. True or False: if False, explain why or provide a counterexample.

(a) A polynomial of the 5th degree can have only 2 real roots and 3 imaginary roots F

(b) A polynomial function of degree 8 can only have as its real roots 2 (m3) and 3 (m2). F

(c) A polynomial function of degree 7 must have at least one rational root. T

(d) A 44th degree polynomial function can have exactly 12 relative extrema. F

(e) Every even degree function is even. F

(f) Every odd polynomial function is also of odd degree. T

(g) An odd degree polynomial has a range of all real numbers. T

(h) An even degree polynomial has a domain of all real numbers. T

(i) Precalculus is awesome. TRUE!