Name_____ Date_____ Period_____

Worksheet 3.1—Polynomial Functions

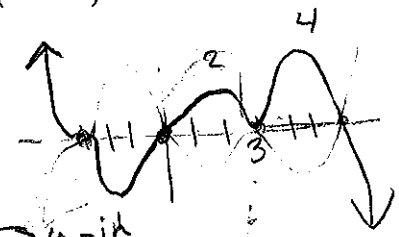
Show all work on a separate sheet of paper. Give simplified, exact values for all answers. **No Calculator is Permitted unless specifically stated.**

I. Multiple Choice

1. Which of the following functions is NOT a polynomial?

(A) $f(x)=\dfrac{-2.3x^4-6x+11}{4}$ (B) $n(t)=5t^2+t^{-1}+3$ (C) $P(x)=\pi x(x^2-ex)$

(D) $y=5$ (E) $h(x)=2.6x-7.7x^3+\sqrt{2}x$

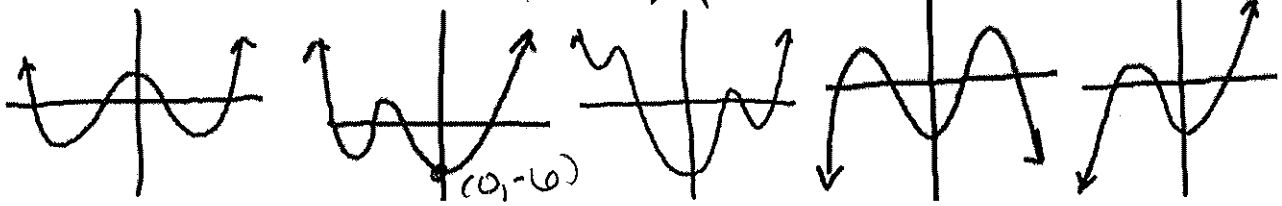


2. The function $f(x)=-2x(x-3)^2(x+3)^3(x-6)$ has how many relative extrema?

(A) 7 (B) 6 (C) 5 (D) 4 (E) 3

3. Which of the following graphs could be the graph of $f(x)=5x^3-5x+5x^2+x^4-6$?

(A) (B) (C) (D) (E)



4. Which of the following statements about a polynomial function with degree n is false?

(A) has at most n turning points (B) may have up to n distinct roots

(C) if n is odd, it has at least one root (D) if n is even, it may have no roots (E) all statements are true

5. A function whose only roots are $x=1\,(m2)$, $x=-2\,(m3)$, and $x=3\,(m1)$, that passes through the point $(-1,-2)$ has a y-intercept of what?

$f(x)=A(x-1)^2(x+2)^3(x-3)$

$-2=A(-1-1)^2(-1+2)^3(-1-3)$

$-2=A(-2)^2(1)^3(-4)$

$-2=A(4)(1)(-4)$

$A=\dfrac{-2}{-16}$

(A) 24 (B) -24 (C) $\dfrac{1}{8}$ (D) 2 (E) -3

II. Short Answer

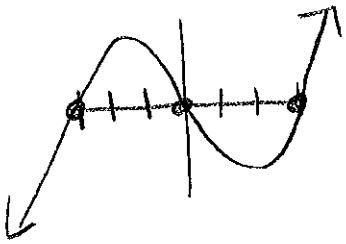
6. Find the roots (by factoring) and both end behaviors for each of the following polynomials. Graph each function.

(a) $f(x)=-\frac{1}{2}x(18-2x^2)$ (b) $P(x)=x^4-2x^3+8x-16$

a) $f(x) = -\frac{1}{2}x(-2x^2+18)$

$= -\frac{1}{2}x(-2(x^2-9)$

$= x(x+3)(x-3)$



$x=0$
$x=-3$
$x=3$

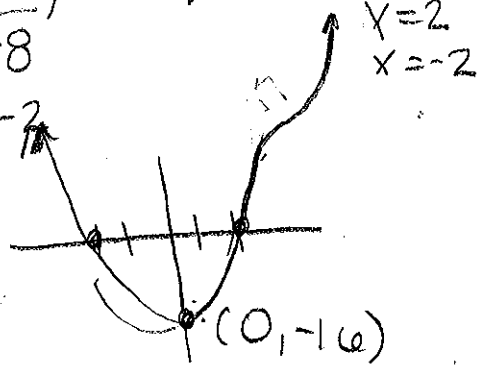
$\lim_{x\to\infty} = \infty$ $\lim_{x\to-\infty} = -\infty$

b) $P(x) = x^3(x-2)+8(x-2)$

$=(x^3+8)(x-2)$

$\sqrt[3]{x^3} = \sqrt[3]{-8}$

$x=-2$



$x=2$
$x=-2$

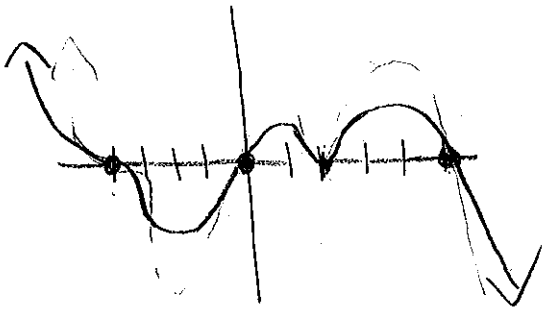
$\lim_{x\to\infty} = \infty$ $\lim_{x\to-\infty} = -\infty$

7. Sketch the following functions. Be sure to clearly show the roots and the multiplicities at each root.

(a) $f(x)=-3x(x-5)(x+4)^3(x-2)^2$ (b) $h(x)=\frac{2}{3}(x-2)^2(x+2)^2(x-5)(x+5)^3$

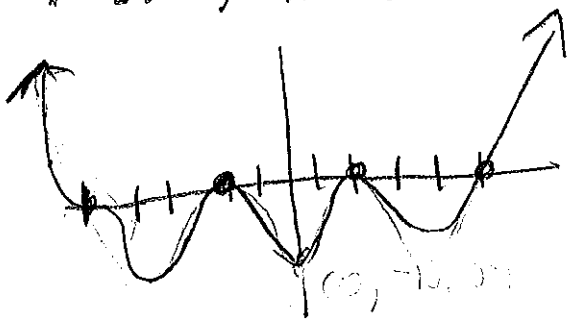
$x=0(m1)$ $x=5(m1)$ $x=-4(m3)$

$x=2(m2)$



$x=2(m2)$ $x=-2(m2)$

$x=5(m1)$ $x=-5(m3)$



8. Write a (a) general equation in factored form of a polynomial whose only roots are $x=3\,(m2)$, $x=-4$ $(m1)$, and $x=0\,(m3)$ and (b) a particular equation if the same polynomial passes through $(-2,2)$

a) $f(x)=Ax^3(x-3)^2(x+4)$

b) $2=A(-2)^3(-2-3)^2(-2+2)$

$2=A(-8)(25)(2)$

$2=A(-400)$

$A=\frac{-1}{200}$

$f(x)=\frac{-1}{200}x^3(x-3)^2(x+4)$

9. Write an equation in factored form of a cubic polynomial, f, with the following characteristics: $f(-5)=f(1)=f(6)=0$, $f(-1)=-3$

$f(x)=A(x+5)(x-1)(x-6)$

$-3=A(-1+5)(-1-1)(-1-6)$

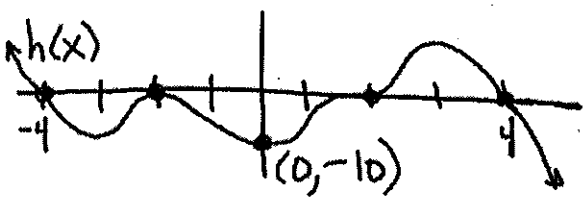
$-3=A(4)(-2)(-7)$

$-3=A(56)$

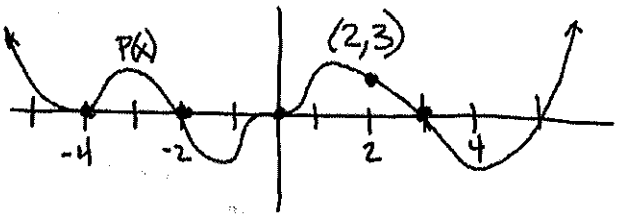
$A=\frac{-3}{56}$ $f(x)=\frac{-3}{56}(x+5)(x-1)(x-6)$

10. Write both a general and particular equation, in factored form, of the polynomial whose graph is given below.

(a)



(b)



(a) $h(x) = A(x+4)(x+2)^2(x-2)^3(x-4)$

$-10 = A(4)(2)^2(-2)^3(-4)$

$-10 = A(4 \cdot 4 \cdot -8 \cdot -4)$

$\frac{-10}{512} = \frac{A(512)}{512}$

$A = \frac{-5}{256}$

$h(x) = \frac{-5}{256}(x+4)(x+2)^2(x-2)^3(x-4)$

(b) $P(x) = Ax^3(x+4)^2(x+2)(x-3)(x-5)$

$3 = A(2)^3(6)^2(4)(-1)(-3)$

$3 = A(8 \cdot 36 \cdot 4 \cdot -1 \cdot -3)$

$\frac{3}{3456} = \frac{A(3456)}{3456}$

$P(x) = \frac{3}{3456}x^3(x+4)^2(x+2)(x-3)(x-5)$

11. (Calculator Permitted) Find all the zeros and relative extrema of the function. List the open intervals of increasing and decreasing. $f(x) = x^4 + 0.1x^3 - 6.5x^2 + 7.9x - 2.4$

Zeros: $x = -3.1, .5, 1.33, 1.367$

Extrema: Rel min $(-2.086, -29.137)$, Rel max $(.751, .207)$, Rel min $(1.26, -.049)$

Increasing: $(-2.086, .751) \cup (1.26, \infty)$

Decreasing: $(-\infty, -2.086) \cup (.751, 1.26)$

12. Which of the following statements are true regarding the graph of the cubic polynomial $f(x) = x^3 + bx^2 + cx + d$? If the statements are false, explain why.

a. It intersects the y-axis in one and only one point. T
b. It intersects the x-axis in at most three points. T
c. It intersects the x-axis at least once. T
d. For $|x|$ very large, it behaves like the graph of $y = x^3$ T
e. It is symmetric with respect to the origin. F
f. It passes through the origin. F
g. It has at least two relative extrema. F