Name Key Date______ Period______

Worksheet 2.4—Parent Functions & Transformations

Show all work on a separate sheet of paper. Give simplified, exact values for all answers. **No Calculator is Permitted unless specifically stated.**

I. Multiple Choice

B 1. Give a function *f*, which of the following represents a horizontal stretch by a factor of 3?

(A) $y = f(3x)$ (B) $y = f\left(\frac{1}{3}x\right)$ (C) $y = 3f(x)$ (D) $y = \frac{1}{3}f(x)$ (E) $y = f(x) + 3$

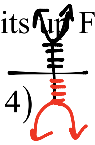
D 2. Give a function *f*, which of the following represents a vertical compression by a factor of 3?

(A) $y = f(3x)$ (B) $y = f\left(\frac{1}{3}x\right)$ (C) $y = 3f(x)$ (D) $y = \frac{1}{3}f(x)$ (E) $y = f(x) + 3$

A 3. Give a function *f*, which of the following represents a horizontal shift 3 units right?

(A) $y = f(x-3)$ (B) $y = f(x+3)$ (C) $y = 3 + f(x)$ (D) $y = f(x) - 3$ (E) $y = f(3x)$

A 4. Give a function *f*, which of the following represents a vertical shift 4 units up FOLLOWED BY a reflection across the *x*-axis?? Be careful! Stated in a different order!



[✓ (A)] $y = -f(x) - 4$ (B) $y = -f(x) + 4$ (C) $y = f(4-x)$ (D) $y = f(x-4)$ (E) $y = -f(x-4)$

↑ Reflected followed by shift up.

B 5. If $f(x) = 2 + \ln\left(3x - \frac{\pi}{2}\right)$, then compared to the parent function $y = \ln x$, the graph of *f* is shifted

$f(x) = \ln 3(x - \frac{\pi}{6}) + 2$

(A) $\pi/2$ units right (B) $\pi/6$ units right (C) 2 units left (D) 3 units left (E) $\pi/2$ units left

A 6. The average rate of change for $f(x) = 1 + \sqrt{x}$ on the interval $[1, 4]$ is

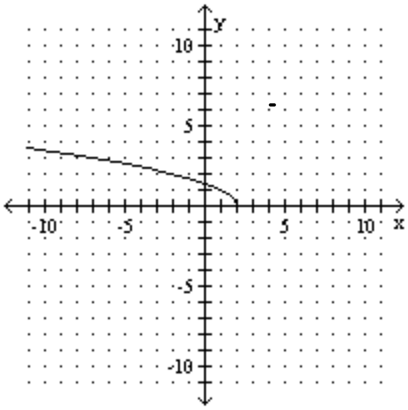
(A) 1/3 (B) 1/2 (C) 0 (D) 2/3 (E) 3/2

$\frac{f(4) - f(1)}{4 - 1}$

$\frac{3 - 2}{3}$

$\frac{1}{3}$

E 7. The graph of a function $f(x)$ is given below. What is the equation of this graph?



(A) $f(x) = \sqrt{-x} + 2$ (B) $f(x) = -\sqrt{x} + 2$ (C) $f(x) = -\sqrt{x+2}$

(D) $f(x) = \sqrt{-x-2}$ $= \sqrt{-(x+2)}$ (E) $f(x) = \sqrt{-x+2}$ $= \sqrt{-(x-2)}$

II. Short Answer

8. Find the domain of each function, put each of the following in standard transformation form, then sketch the graph showing important information. Compare your algebraic domain to the domain of your graph.

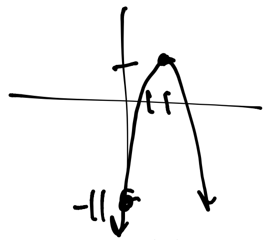
(a) $f(x) = -3(x-2)^2 + 1$

$D_f : \mathbb{R}$

- Reflects across the x axis
- Verticle stretch bfo 3
- Shifts Right 2 units
- Shifts up 1 unit

$f(0) = -3(-2)^2 + 1$

$= -11$



(b) $f(x) = \frac{1}{2}\sqrt{8x+4} - 3$

Domain $f(x) = \frac{1}{2}\sqrt{8(x+\frac{1}{2})} - 3$

$8x+4 \geq 0$

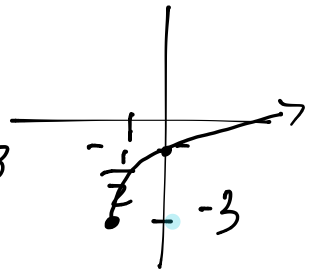
$8x \geq -4$

$x \geq -\frac{1}{2}$

$D_f : \{x \mid x \geq -\frac{1}{2}\}$

$f(0) = \frac{1}{2}\sqrt{4} - 3$

$= -1$



Vertical compression bio 2
Horizontal compression bfo 8
Shifts left 1/2 unit
Shits down 3 units

(c) $f(x) = 2 - \frac{3}{2x-5}$

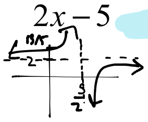
$f(x) = \frac{-3}{2(x-\frac{5}{2})} + 2$

$= -\frac{3}{2}\left(\frac{1}{x-\frac{5}{2}}\right) + 2$

$2x-5 \neq 0$

$x \neq 5/2$

$D_f : \{x \mid x = 5/2\}$



$f(0) = 2 + \frac{3}{5}$

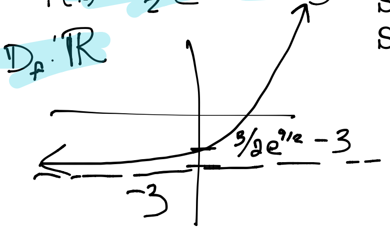
$= \frac{13}{5}$

Reflects across the x-axis
Vertical stretch bfo 3/2
Shifts right 5 units
Shifts up 2 units

(d) $f(x) = \frac{3}{2}e^{\frac{2}{3}x - \frac{9}{2}} - 3$

$f(x) = \frac{3}{2}e^{2/3(x-\frac{27}{4})} - 3$

$D_f : \mathbb{R}$



Vertical stretch bio 3/2
Horizontal stretch bfo3/2
Shifts right 27/4 units
Shifts down 3 units

$f(0) = \frac{3}{2}e^{-9/2} - 3$

$= \frac{3}{2e^{9/2}} - 3$

(e) $f(x) = \frac{2x-3}{4x-1}$

$f(0) = \frac{-3}{-1}$

$= 3$

$$4x-1 \overline{\smash{)}2x-3} \quad \frac{1}{2}$$

$-2x + \frac{1}{2}$

$-\frac{5}{2}$

$f(x) = \frac{1}{2} - \frac{5/2}{4x-1}$

$= \frac{1}{2} - \frac{5}{2(4x-1)}$

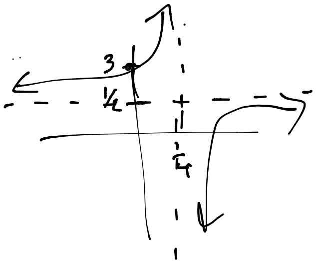
$= \frac{-5}{2 \cdot 4(x-\frac{1}{4})} + \frac{1}{2}$

$= \frac{-5}{8(x-\frac{1}{4})} + \frac{1}{2}$

$= -\frac{5}{8} \cdot \frac{1}{x-\frac{1}{4}} + \frac{1}{2}$

Domain
$4x-1 \neq 0$
$x \neq \frac{1}{4}$

$D_f : \{x \mid x \neq \frac{1}{4}\}$



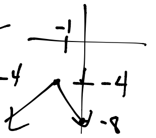
Reflects across the x-axis
Vertical compression bfo 8/5
shifts right 1/4 unit
Shifts up 1/2 unit

(f) $f(x) = -2 - 2|-2x-2| - 2$

$f(x) = -2|-2(x+1)| - 4$

$D_f : \mathbb{R}$

$f(0) = -2|-2| - 4$
$= -4 - 4$
$= -8$



Reflects across the x-axis
Vertical stretch bfo 2
Reflects across the x-axis
(however, it's an even function)
Horizontal compression
bfo shifts left 1 unit
Shifts down 4 units

9. If $g(x) = f(ex)$ is a transformation of a function $y = f(x)$, by algebraically manipulating each function, describe TWO different ways, if possible, to obtain the graph of g from the graph of f by a standard transformation. Note: $e \approx 2.718$.

(a) $f(x) = x$
$g(x) = (ex)$ or $g(x) = e(x)$
Horizontal compression bfo e
Vertical stretch bfo e

(b) $f(x) = x^2$
$g(x) = (ex)^2$ or $g(x) = e^2(x)^2$
Horizontal compression bfo e
Vertical stretch bfo e^2

(c) $f(x) = x^3$
$g(x) = (ex)^3$ or $g(x) = e^3(x)^3$
Horizontal compression bfo e
Vertical stretch bfo e^3

(d) $f(x) = \sqrt{x}$
$g(x) = \sqrt{ex}$ or $g(x) = \sqrt{e} \cdot \sqrt{x}$
Horizontal compression bfo e
Vertical stretch bfo $e^{1/2}$

(e) $f(x) = x^{-1}$
$g(x) = \frac{1}{ex}$ or $g(x) = \frac{1}{e} \cdot \frac{1}{x}$
Horizontal compression bfo e
Vertical compression bfo e

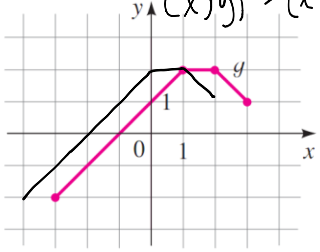
(f) $f(x) = |x|$
$g(x) = |e \cdot x|$ or $g(x) = e|x|$
Horizontal compression bfo e
Vertical stretch bfo e

(g) $f(x) = \ln x$
$g(x) = \ln(ex)$ or $g(x) = \ln e + \ln x$
$= \ln x + \ln e$
$= \ln x + 1$
Horizontal compression bfo e
Shifts up one unit

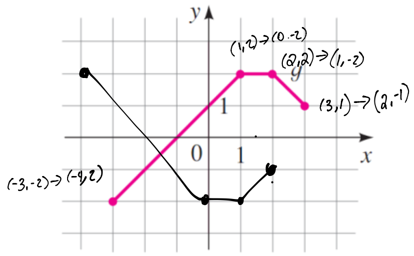
(h) $f(x) = e^x$
$g(x) = e^{ex}$
no other way to obtain the graph
Horizontal compression bfo e

The graph of h is given at right. Sketch the graphs of the following functions. Be sure to scale your graphs:

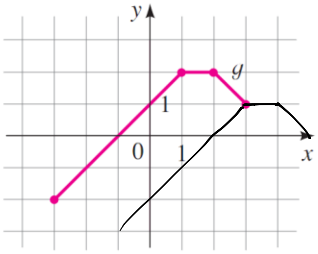
(a) $y = h(x+1)$ $(x, y) \to (x-1, y)$



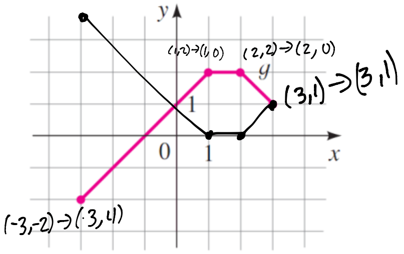
(b) $y = -h(x+1)$ $(x, y) \to (x-1, -y)$



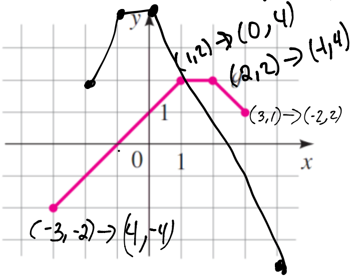
(c) $y = h(x-2)-1$ $(x, y) \to (x+2, y-1)$



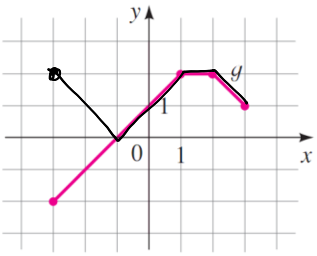
(d) $y = -h(x)+2$ $(x, y) \to (x, -y+2)$



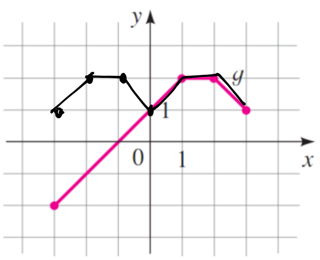
(e) $y = 2h(1-x)$ $y = 2h(-x+1)$
$= 2h(-(x-1))$
$(x, y) \to (-x+1, 2y)$



(f) $y = |h(x)|$ $(x, y) \to (x, |y|)$



(g) $y = h(|x|)$



(h) $y = |h(|x|)|$