Name KEY Date______________________ Period______

Worksheet 2.3—Other Properties of Functions

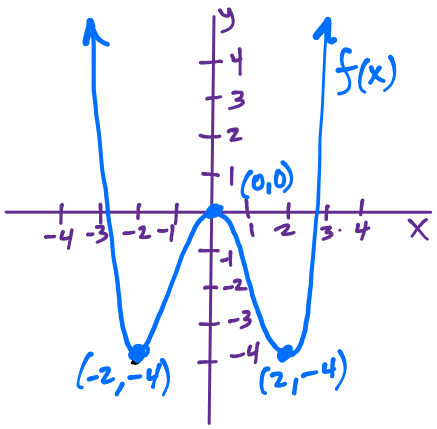
Show all work on a separate sheet of paper. Give simplified, exact values for all answers. **No Calculator is Permitted unless specifically stated.**

I. Multiple Choice

1. Which of the following scenarios of functions is continuous?
   (A) The number of students enrolled at New Braunfels High School at any given time
   (B) The outdoor temperature as a function of time
   (C) The cost of a U.S. postage stamp as a function of the weight of the letter
   (D) The price of a stock as a function of time
   (E) Number of drinks sold at an outdoor event as a function of the outdoor temperature

2. Which of the following functions is decreasing? **(Calculator OK—Ha!)**
   (A) The outdoor temperature as a function of time
   (B) The Dow Jones Industrial average as a function of time
   (C) Air pressure in the Earth's atmosphere as a function of altitude
   (D) World population since 1900 as a function of time
   (E) Water pressure in the ocean as a function of depth

Use the graph below to answer questions 3 and 4



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3. Estimate the local extrema.
   (A) Local Max of 0; Local Min of -4 (B) No Local Max; Local Min of -4 (C) No Local Extrema
   (D) Local Max of 0; Local Mins ~~of~~ at $-2, 2$ (E) Local Max of ∞; Local Mins of $-2, 2$

4. Find the open intervals of increasing/decreasing/constant
   (A) Inc: $(0,\infty)$; Dec: $(-4,0)$ (B) Inc: $(-2,2)$; Dec: $(-\infty,-2)\cup(2,\infty)$
   (C) Inc: $(-2,0)\cup(2,\infty)$; Dec: $(-\infty,-2)\cup(0,2)$ (D) Inc: $(0,\infty)$; Dec: $(-\infty,0)$ (E) Inc: $(-\infty,\infty)$

5. $\displaystyle\lim_{x\to-\infty}\frac{-2x^3+3x-4}{x^2-4}=$

   (A) 1 (B) -2 (C) 0 (D) ∞ (E) $-\infty$

End behavior will look like $y=-x$

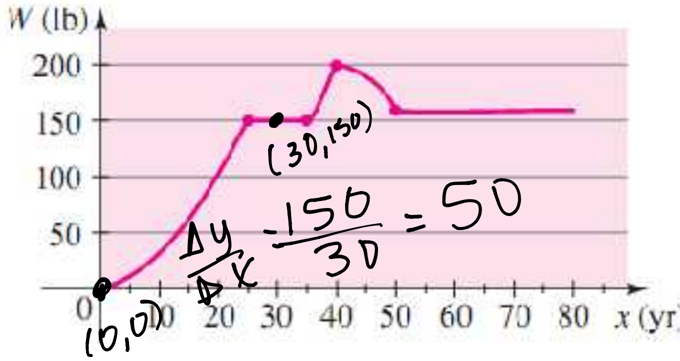
6. Which of the following is NOT true about the function $f(x)=\frac{3x^2-27}{x^2-25}$ $=\frac{3(x^2-9)}{(x+5)(x-5)}=\frac{3(x+3)(x-3)}{(x+5)(x-5)}$

~~(A)~~ It is an even function ~~(B)~~ It has a HA at $x=3$ ~~(C)~~ It has VA's at $x=-5,\ x=5$

~~(D)~~ It has x-intercepts at $x=-3,\ x=3$ (E) It has a Local Max at $\left(0,\frac{27}{25}\right)$

7. Grandpa George is still 80 years old and is still reminiscing about how his weight has fluctuated over his years. He envisions the following piece-wise graph in his head. What is Grandpa's average yearly weight gain from the day of his birth to his **30**th birthday (in pounds per year)

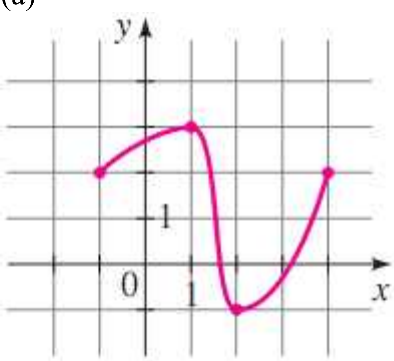
(A) 200 (B) 150 (C) 100 (D) 50 (E) 5



II. Short Answer

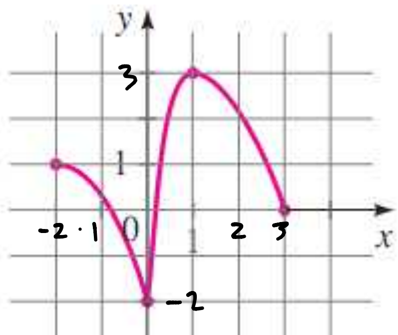
8. For each graph below, determine and label (i) domain and range, (ii) the open intervals of increasing/decreasing/constant, and (iii) coordinates of relative extrema. Each graph shows the function in its entirety.

(a)



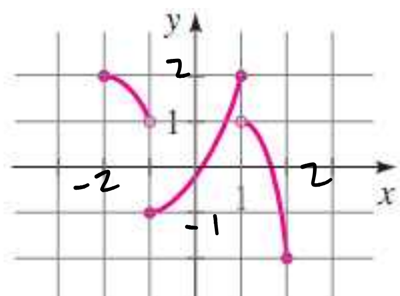
D: [-1,4], R: [-1,3]
Max of y = 3 at x = 1
Min of y = -1 at x = 2
Local max of y = 3 at x = 1
Local min of y = -1 at x = 2

(b)



D: [-2,3], R: [-2,3]
Max of y = 3 at x = 1
Min of y = -2 at x = 0
Local max of y = 3 at x = 1
Local min of y = -2 at x = 0

(c) (this one has tricky extrema)



D: [-2,2], R: [-2,2]
Max of y = 2 at x = -2 & x = 1
Min of y = -2 at x = 2
Local max of y = 2 at x = 1
Local min of y = -1 at x = -1

9. **(Calculator OK)** To three decimals, for each of the following, carefully and deliberately, find and label each of the following: (i) Domain and Range, (ii) Coordinates of any Local Extrema, and (iii) Open intervals of increasing/decreasing/constant. Don't list any symmetries! None of these will have any!!

(a) $f(x)=x^3+2x^2-x-2$ (b) $g(x)=x^4-4x^3+2x^2+4x-3$ (c) $f(x)=\frac{-3x}{2x-3}$

(a)
i) D_f: ℝ
ii) local min of y = -3.052 at x = -2.553
local max of y = .631 at x = -1.548
local min of y = -2.112 at x = .215
iii) Inc $(-\infty,-2.553)\cup(.215,\infty)$
Dec $(-2.553,.215)$

(b)
i) D_g: ℝ
ii) local min of y = -4 at x = -.414
local max of y = 0 at x = 1
local min of y = -4 at x = 2.414
iii) Inc: $(-.414,1)\cup(2.414,\infty)$
Dec: $(-\infty,-.414)\cup(1,2.414)$

(c)
$2x-3\neq 0$
$x\neq\frac{3}{2}$
i) D_f: $\{x \mid x\neq\frac{3}{2}\}$
ii) no local Extremum
iii) Inc: $(-\infty,\frac{3}{2})\cup(\frac{3}{2},\infty)$
Monotonic Increasing over it's domain

10. For the following functions, find the average rate of change on the indicated interval. Be sure to show the **difference quotient.**

(a) $f(x) = 3x - 2,\ x \in [2,3]$

$$\frac{f(3)-f(2)}{3-2} = \frac{7-4}{1}$$

$$= 3$$

(b) $f(z) = 1 - 3z^2,\ -2 \le z \le 0$

$$\text{Avg ROC} = \frac{f(0)-f(-2)}{0-(-2)}$$

$$= \frac{[1-3\cdot 0^2]-[1-3(-2)^2]}{0+2}$$

$$= \frac{1-(-11)}{2}$$

$$= \frac{12}{2}$$

$$= 6$$

(c) $g(t) = \dfrac{2}{t+1},\ t \in [0,5]$

$$\frac{g(5)-g(0)}{5-0} = \frac{\frac{2}{6}-\frac{2}{1}}{5}$$

$$= \frac{\frac{1}{3}-2}{5}\cdot\frac{3}{3}$$

$$= \frac{1-6}{15}$$

$$= \frac{-5}{15}$$

$$= \frac{-1}{3}$$

(d) $f(x) = x^2 + x - 2,\ c \le x \le c+h$

$$\frac{f(c+h)-f(c)}{c+h-c} = \frac{(c+h)^2+(c+h)-2-(c^2+c-2)}{h}$$

$$= \frac{c^2+2ch+h^2+c+h-2-c^2-c+2}{h}$$

$$= \frac{2ch+h^2+h}{h}$$

$$= \frac{h(2c+h+1)}{h}$$

$$= 2c+h+1$$

11. A student being chased by a math teacher trying to assign him math homework is running around the track out by the football field. The math teacher records the blazing fast student's time every half lap (200 m), obtaining the data below.

Time (s)	Distance (m)	
32	200	1
68	400	
108	600	2
152	800	
203	1000	3
263	1200	
335	1400	4
412	1600	

(a) What was the student's average speed between 68 s and 152 s?

$$\text{Avg speed} = \frac{D(152)-D(68)}{152-68} = \frac{800-400}{84}$$

$$= \frac{400}{84} = \frac{100}{21} \text{ meters per second}$$

(b) What was the student's average speed between 263 s and 412 s?

$$\text{Avg Speed} = \frac{D(412)-D(263)}{412-263} = \frac{1600-1200}{149}$$

$$= \frac{400}{149} \text{ meters per second}$$

(c) Calculate the students average speed for each lap (not half lap). Is the student speeding up, slowing down, or neither. Show the calculations that lead to your answer.

Speed of lap 1 $= \frac{400}{68}$ m/s
Speed of lap 2 $= \frac{400}{152-68} = \frac{400}{84}$ m/s
Speed of lap 3 $= \frac{400}{263-152} = \frac{400}{111}$ m/s
Speed of lap 4 $= \frac{400}{412-263} = \frac{400}{149}$ m/s

1st lap 68 sec
2nd lap 84 sec
3rd lap 111 sec
4th lap 149 sec

Speed is slowly down

(d) **Genius question**: Assuming the student is running around the track counterclockwise and the astute math teacher runs clockwise to catch him, when they are 40 m from each other, how does the Coriolis Effect affect the teacher's aim if he were to throw the homework assignment to his student? Assume these shenanigans are taking place here in the Northern Hemisphere.

He will have to lead the runner a bit to account for the rotation.

12. Determine algebraically if each of the following are even, odd, or neither.

(a) $f(x) = 4x^5 - 3x^3 + x - 1$

$f(-x) = 4(-x)^5 - 3(-x)^3 + (-x) - 1$

$= -4x^5 + 3x^3 - x - 1$

neither

(b) $g(x) = -6\sqrt[5]{6x}$

$g(-x) = -6\sqrt[5]{6(-x)}$

$= 6\sqrt[5]{6x}$

ODD

(c) $h(j) = \sqrt{2j - 1}$

$h(-j) = \sqrt{2(-j) - 1}$

$= \sqrt{-2j - 1}$

neither

(d) $d(y) = \dfrac{y^2 - \pi}{3y - \sqrt[3]{2y}}$

$d(-y) = \dfrac{(-y)^2 - \pi}{3(-y) - \sqrt[3]{2(-y)}}$

$= \dfrac{y^2 - \pi}{-3y + \sqrt[3]{2y}}$ $\dfrac{\text{even}}{\text{odd}} = \text{odd}$

(e) $k(v) = \dfrac{2v - 1}{7v^2 - 6v + 4}$

$k(-v) = \dfrac{2(-v) - 1}{7(-v)^2 - 6(-v) + 4}$

$= \dfrac{-2v - 1}{7v^2 + 6v + 4}$ $\dfrac{\text{neither}}{\text{neither}} = \text{neither}$

(f) $s(w) = \dfrac{w^{4/5}}{w^{59} - 16w^{33}}$

$s(-w) = \dfrac{(-w)^{4/5}}{(-w)^{59} - 16(-w)^{33}}$

$= \dfrac{w^{4/5}}{-w^{59} + 16w^{33}}$ $\dfrac{\text{even}}{\text{odd}} = \text{odd}$

13. For each of the following, find the domain, end behavior, x-intercepts, y-intercepts, any symmetry, and find and label all discontinuities. Use your collected information to sketch a graph.

(a) $f(x) = \dfrac{-8x^2 - 10x - 3}{2x^2 + x}$

(b) $g(t) = \dfrac{3}{t^2 - 25}$

(c) $p(x) = \begin{cases} \dfrac{x^2 + x - 2}{x - 1}, & x \neq 1 \\ 3, & x = 1 \end{cases}$

next page

$f(x) = \frac{-(8x^2+10x+3)}{x(2x+1)}$

$= \frac{-(4x+3)(2x+1)}{x(2x+1)}$

$x \neq 0 \quad 2x+1 \neq 0$

$x \neq -1/2$

$D_f: \{x \mid x \neq 0, -\frac{1}{2})$

VA @ $x = 0$

HA @ $y = -4$

$\lim_{x \to \infty} f(x) = -4$

$\lim_{x \to -\infty} f(x) = -4$

hole $y = \frac{-(4(-\frac{1}{2})+3)}{-\frac{1}{2}}$

$= \frac{-(-2+3)}{-\frac{1}{2}}$

$= \frac{-1}{-\frac{1}{2}}$

hole @ $(-\frac{1}{2}, 2)$

y int: none $x \neq 0$

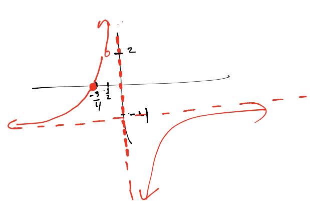
$4x+3=0$ x-int

$x = -3/4$

x int $(-\frac{3}{4}, 0)$

Symmetry: none

$f(-x) = \frac{-8x^2+10x-3}{2x^2-x}$



$g(t) = \frac{3}{(t+5)(t-5)}$

$D_g: \{t \mid t \neq -5, 5\}$

VA $t=-5, t=5$

HA $y=0$

no holes

$\lim_{x \to \infty} g(t) = 0$

$\lim_{x \to -\infty} g(t) = 0$

Symmetry

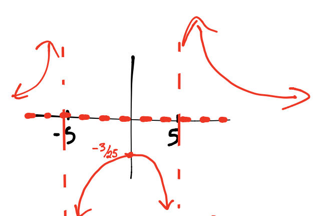
$g(-t) = \frac{3}{t^2-25}$

Even function

$g(0) = \frac{3}{-25}$

y int $(0, -3/25)$

x int: none



$g(6) = \frac{3}{36-25}$ (pos #)

$g(-6) = \frac{3}{36-25}$ (pos #)

$P(x) = \begin{cases} \frac{(x+2)(x-1)}{(x-1)}, & x \neq 1 \\ 3 & x = 1 \end{cases}$

D: $\mathbb{R}$

y int:

$y = 0+2$

$y = 2$

$(0, 2)$

x int

$0 = x+2$

$x = -2$

$(-2, 0)$