

Name \_\_\_\_\_ Date \_\_\_\_\_ Period \_\_\_\_\_

**Worksheet 7.3—Plane Curves & Parametric Equations**Show all work on a separate sheet of paper. **No Calculator** unless otherwise specified.**Multiple Choice**

1. Which of the following points corresponds to  $t = -1$  in the parameterization  $x = t^2 - 4$ ,  $y = t + \frac{1}{t}$ ?

(A)  $(-3, -2)$       (B)  $(-3, 0)$       (C)  $(-5, -2)$       (D)  $(-5, 0)$       (E)  $(3, 2)$

2. Which of the following values of  $t$  produces the same point as  $t = \frac{2\pi}{3}$  in the parametrization  $x = 2 \cos t$ ,  $y = 2 \sin t$ ?

(A)  $t = -\frac{4\pi}{3}$       (B)  $t = -\frac{2\pi}{3}$       (C)  $t = -\frac{\pi}{3}$       (D)  $t = \frac{4\pi}{3}$       (E)  $t = \frac{7\pi}{3}$

3. A rock is thrown straight up from level ground with its position above ground at any time  $t \geq 0$  given by  $x = 5$ ,  $y = -16t^2 + 80t + 7$ . At what time will the rock be 91 feet above ground?

(A) 1.5 sec      (B) 2.5 sec      (C) 3.5 sec      (D) 1.5 sec and 3.5      (E) NEVER

4. Which of the following describes the graph of the parametric equations  $x = 1 - t$ ,  $y = 3t + 2$ ,  $t \geq 0$ ?

(A) a straight line      (B) a line segment      (C) a ray      (D) a parabola      (E) a circle

**Short Answer**

(Calculator Permitted) For each of the following 5 – 10, a pair of parametric equations is given. (a) sketch the curve represented by the parametric equations showing the path and direction, then (b) find a rectangular-coordinate equation for the curve by eliminating the parameter.

5.  $x = 2t$ ,  $y = t + 6$

6.  $x = t^2$ ,  $y = t - 2$ ,  $2 \leq t \leq 4$

7.  $x = t + 1$ ,  $y = \frac{t}{t + 1}$

8.  $x = 3 \cos t$ ,  $y = 5 \sin t$ ,  $0 \leq t \leq 2\pi$

9.  $x = \sin^2 t$ ,  $y = \cos t$

10.  $x = \tan t$ ,  $y = \cot t$ ,  $0 < t < \frac{\pi}{2}$

For problems 11 & 12, find parametric equations for the line with the given properties.

11. slope of  $\frac{1}{2}$ , passing through  $(4, -1)$

12. passing through  $(6, 9)$  and  $(9, 12)$

13. Find parametric equations for the circle  $x^2 + y^2 = r^2$ , centered at the origin with radius of  $r$ .
14. (Calculator permitted) Suppose a gun fires a bullet into the air from a height of 5 feet with an initial speed of 2048 ft/s at an angle of  $30^\circ$  to the horizontal.
- (a) After how many seconds will the bullet hit the ground?
- (b) How far from the gun will the bullet hit the ground?
- (c) What is the maximum height attained by the bullet?

For 15 – 17, using your graphing calculator, sketch the curve represented by the following parametric equations. Transfer the graph to your paper showing the path and direction.

15.  $x = \sin t$ ,  $y = 2 \cos 3t$

16.  $\sin(\cos t)$ ,  $y = \cos(t^{3/2})$ ,  $0 \leq t \leq 2\pi$

17.  $x = 2 \cos t + \cos 2t$ ,  $y = 2 \sin t - \sin 2t$

For 18 & 19, a polar equation is given. (a) express the polar equation in parametric form, then (b) use your graphing calculator to graph the parametric equations. Transfer the graph to your paper showing the direction and path.

18.  $r = 2^{\theta/12}$ ,  $0 \leq \theta \leq 4\pi$

19.  $r = \sin \theta + 2 \cos \theta$

20. (Calculator permitted) The curves  $A$ ,  $B$ ,  $C$ , and  $D$  are defined parametrically as follows, where the parameter  $t$  takes on all real values unless stated otherwise:

$$A: x = t, y = t^2$$

$$B: x = \sqrt{t}, y = t, t \geq 0$$

$$C: x = \sin t, y = \sin^2 t$$

$$D: x = 3^t, y = 3^{2t}$$

- (a) Show that the points on all four of these curves satisfy the same rectangular coordinate equation.

- (b) Draw the graph of each curve and explain how the curves differ from each other. Be sure to address path, speed, and direction.

21. (Calculator Permitted) Archimedes hits a baseball when it is 4 feet above the ground with an initial velocity of 120 feet per second. The ball leaves the bat at a  $30^\circ$  angle with the horizontal and heads toward a 30-foot fence 350 feet from home plate.

(a) Write a set of parametric equations representing the height of the ball and its distance from home plate.

(b) If so, by how much does the baseball clear the fence? If not, could the ball be caught? Justify.

(c) suppose that the moment Archimedes hits the ball, there is a 5-foot per second wind gust. Assuming that this horizontal wind gust acts in the horizontal direction out with the ball, does the ball clear the fence? If so, by how much? If not, could the ball be caught? Justify.