Name_____ Date_____ Period_____

Xtra Practice: Proofs and Trig equations

Part I: Prove the following identities. Show all steps working down vertically. Each line must be an equivalent statement to the one above it.

1. $1+\dfrac{\cot A}{\csc A}-\sin^2 A=\cos A(\cos A+1)$

2. $\tan B+\cot B=\csc B\sec B$

3. $\dfrac{\cot\phi}{\sec\phi}=\csc\phi-\sin\phi$

4. $\dfrac{\sec\beta}{1+\cos\beta}=\csc^2\beta(\sec\beta-1)$

5. $(\tan\alpha-\sec\alpha)^2=\dfrac{1-\sin\alpha}{1+\sin\alpha}$

6. $\sin^4\psi-\cos^4\psi=1-2\cos^2\psi$

7. $\sec^4\delta - 2\sec^2\delta\tan^2\delta + \tan^4\delta = 1$

8. $\sqrt[3]{\tan^2 x - \sec^2 x} = -1$

9. $\dfrac{1}{\csc y + \cot y} = \dfrac{1-\cos y}{\sin y}$

10. $\dfrac{\sin\kappa - 1}{\cos\kappa} = \tan\kappa - \sec\kappa$

11. $\dfrac{\sec\omega}{\sin\omega} - \dfrac{\sin\omega}{\cos\omega} = \cot\omega$

Part II: Solve

12. Find the exact values for $\sqrt{3}\cot x - 1 = 0 \quad x \in [0, 2\pi)$ (hint: turn it into a $\tan x$ problem)

Part III: Review

Determine the exact values of the other five trigonometric functions for θ. Given the following information. Draw the reference triangles.

13. $\cos\theta = -\frac{7}{13}; \frac{\pi}{2} < \theta < \pi$

14. $\sin\theta = -\frac{4}{5}; \frac{3\pi}{2} < \theta < 2\pi$

Part IV:

Determine whether the following equations are true or false. Show the final values you used in making your decision.

15. $\tan^2(-\pi) + \sec\frac{3\pi}{4} = \sqrt{\left[\cos^2\left(\frac{5\pi}{6}\right) + \sin^2\left(\frac{5\pi}{6}\right)\right]}$

16. $\csc\frac{5\pi}{6} + \sec\left(-\frac{\pi}{3}\right) = \left[\cot^2\left(\frac{3\pi}{4}\right) + \tan^2\left(\frac{7\pi}{4}\right)\right]^2$

Part V: Additional Practice:
Prove each identity.

17. $\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

18. $\left(1-\sin^2\theta\right)\left(1+\tan^2\theta\right) = 1$

19. $\dfrac{1}{1+\sin\phi} + \dfrac{1}{1-\sin\phi} = 2\sec^2\phi$

20. $\cos^2\theta - \sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

21. $\left(\csc\beta - \sin\beta\right)^2 = \cot^2\beta - \cos^2\beta$

22. $\dfrac{\cos\theta}{1-\sin\theta} + \dfrac{1-\sin\theta}{\cos\theta} = 2\sec\theta$

23. $\dfrac{\tan^2 x}{\sec x + 1} = \dfrac{1 - \cos x}{\cos x}$

24. $\dfrac{\cot x}{\csc x + 1} = \dfrac{\csc x - 1}{\cot x}$

Part VI: Simplify each expression (as a single trig function without a fraction, if possible)

25. $\sin^2 \theta + \cos 2\theta$

26. $\sec^4 x - \tan^4 x - \tan^2 x$

27. $\dfrac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta}$

28. $\dfrac{1 + \tan \phi}{\sin \phi} - \sec \phi$

Part VII: Find a counterexample to show that each statement is not an identity.

29. $\cos 2\theta = 2\cos\theta$

30. $\sin(\theta - \beta) = \sin\theta - \sin\beta$

31. $\tan\frac{1}{2}\theta = \frac{1}{2}\tan\theta$

32. $\sqrt{\tan^2\theta + 1} = \sec\theta$