Name_____ Date_____ Period_____

Worksheet 4.4—Properties of Logs

Show all work. All answers must be given as either simplified, exact answers. No calculator is permitted unless otherwise stated.

Multiple Choice

1. $\log 12 =$

(A) $3\log 4$ (B) $\log 3 + \log 4$ (C) $4\log 3$ (D) $\log 3 \cdot \log 4$ (E) $2\log 6$

2. $\log_9 64 =$

(A) $5\log_3 2$ (B) $\left(\log_3 8\right)^2$ (C) $\dfrac{\ln 64}{\ln 9}$ (D) $2\log_9 32$ (E) $\dfrac{\log 64}{9}$

3. $2^{-1} \cdot \left(-3\ln 2 - 1\right) =$

(A) $-\dfrac{1}{2}\ln\left(8e\right)$ (B) $-\ln\left(8e\right)$ (C) $-\dfrac{3}{2}\ln 2$ (D) $-\dfrac{1}{2}$ (E) $\dfrac{1}{8}$

4. $\log_{1/2} x^2 =$

(A) $-2\log_2 x$ (B) $2\log_2 x$ (C) $-0.5\log_2 x$ (D) $0.5\log_2 x$ (E) $-2\log_2 |x|$

5. $\ln x^5 =$

(A) $\dfrac{5\log_7 x}{\log_7 e}$ (B) $\dfrac{2\log x^3}{\log e}$ (C) $\dfrac{x\log_{1/2} 5}{\log_{1/2} e}$ (D) $3\ln x^2$ (E) $\ln x^2 \cdot \ln x^3$

Short Answer

6. Evaluate each of the following expressions using the properties of logs (and no calculator).

(a) $\log_3 \sqrt[3]{81}$ (b) $\log 4 + \log 25$ (c) $\log_2 6 - \log_2 15 + \log_2 20$ (d) $\ln\left(\ln e^{e^{200}}\right)$

7. Use the properties of logs to expand the following expressions.

(a) $\log_5 \sqrt[4]{x^3\left(x^2+1\right)}$

(b) $\log_6 \sqrt{\dfrac{5x^2y^3}{x^2+y^3}}$

(c) $\log\sqrt{x\sqrt{y\sqrt{z}}}$

(d) $\ln\left(\dfrac{7x^4\sqrt{x^4-7}}{e^2\left(x-5\right)^2\sqrt[3]{2-6x^2}}\right)$

8. Use the properties of logs to condense the following expressions.

(a) $4\ln x-\dfrac{1}{3}\ln\left(x^2+1\right)+2\ln\left(x-1\right)$

(b) $\dfrac{1}{3}\ln\left(2x+1\right)+\dfrac{1}{2}\left[\log\left(x-4\right)-\log\left(x^4-x^2-1\right)\right]$

(c) $\log\left(x^2-1\right)-\ln\left(x-1\right)$ (use the change of base formula on this one first to get both in terms of base *e*)

9. If $\log_7 x = A\log_{2/3} x$, use the change of base formula to find the value of A,

10. Simplify the following to a single log expression of the form $\log_b a$: $\left(\log_7 3\right)\left(\log_2 5\right)\left(\log_5 7\right)$

11. Use the properties of logs to prove that $-\ln\left(x-\sqrt{x^2-1}\right)=\ln\left(x+\sqrt{x^2-1}\right)$. You may want to eventually multiply be a clever form of one.

12. Let $A=\ln 3$ and $B=\ln 5$, write each of the following in terms of A and/or B.
(a) $\ln 15$ (b) $\ln 27$ (c) $\ln 75$ (d) $\ln 45$ (e) $\log_5\sqrt{27}$

13. (Calculator Permitted) Solve the following equations graphically on your calculator. Be sure to report three decimals in your answers.
(a) $\ln x > \sqrt[3]{x}$ (b) $1.2^x \le \log_{1.2} x$

14. (Calculator Permitted) The "Occupy Wall Street" movement in 2011 is a protest against the unequal distribution of wealth in the United States. Vilfredo Pareto (1848-1923) observed that most of the wealth of any country is owned by a few members of the population. Pareto's Principle (also known as the 80-20 rule, since roughly 80% of outcomes come from 20% of the causes) is given by

$$\log P = \log c - k \log W$$

Where W is the wealth level (how much money a person has) **in millions of dollars**, c is the population of the country, and P is the number of people in the population having that much money.

(a) Solve the equation for P.

(b) If the population of the US is considered to be 312 million people, that is $c = 312{,}000{,}000$, using $k = 2.824$, what percentage of the US population has \$20 million or more (that is, $W = 20$)? **How does that make you feel????**