Name____ Date_______________________ Period______

Worksheet 3.2—Real Zeros of Polynomial Functions

Show all work. Give simplified, exact values for all answers. **No Calculator is permitted unless specifically stated.**

I. Multiple Choice

1. Let *f* be a polynomial function with integer coefficients such that $f\left(\sqrt{3}\right)=0$. Which of the following statements is not necessarily true?
   (A) $x+\sqrt{3}$ is a factor of $f\left(x\right)$ (B) $x-\sqrt{3}$ is a factor of $f\left(x\right)$ (C) $x=\sqrt{3}$ is a root of $f\left(x\right)$
   (D) x^2+3 is a factor of $f\left(x\right)$ (E) x^2-3 is a factor of $f\left(x\right)$

2. If $f\left(x\right)=6x^3+14x^2-25x+4$, then how many possible distinct rational roots does $f\left(x\right)$ have?
   (A) 8 (B) 12 (C) 16 (D) 24 (E) 30

3. Let $f\left(x\right)=\left(x+2\right)\left(x^2+x-1\right)-3$. Which of the following statements is not true?
   (A) The remainder when $f\left(x\right)$ is divided by $x+2$ is -3
   (B) The remainder when $f\left(x\right)$ is divided by $x-2$ is -3
   (C) The remainder when $f\left(x\right)$ is divided by x^2+x-1 is -3
   (D) $x+2$ is not a factor of $f\left(x\right)$
   (E) $f\left(x\right)$ is not evenly divisible by $x+2$

4. Let $f(x)=(x^2+1)(x-2)+7$. Which of the following statements is not true?
   (A) The remainder when $f(x)$ is divided by x^2+1 is 7
   (B) The remainder when $f(x)$ is divided by $x-2$ is 7
   (C) $f(2)=7$
   (D) $f(0)=5$
   (E) f does not have a real root

5. A degree 3 polynomial with integer coefficients with roots $x=1$ and $x=4-2\sqrt{3}$ that passes through the point $(-1,-52)$ has a *y*-intercept of what?

   (A) -8 (B) -52 (C) 2 (D) $\frac{1}{2}$ (E) 4

II. Short Answer

6. Two polynomials are given. Use either synthetic or long division to divide $P(x)$ by $D(x)$. Express your answer in two ways: $P(x)=D(x)\cdot Q(x)+R(x)$ and $\dfrac{P(x)}{D(x)}=Q(x)+\dfrac{R(x)}{D(x)}$.

   (a) $P(x)=x^3+4x^2-6x+1,\ D(x)=x-1$ (b) $P(x)=4x^3+7x+9,\ D(x)=2x+1$

7. Use synthetic division to find the simplified, exact zeroes of each of the following. Use the quadratic formula if necessary.

   (a) $f(x)=2x^3+x^2-41x+20$, if $f\left(\dfrac{1}{2}\right)=0$ (b) $f(x)=x^4+3x^3-16x^2-27x+63$, if $f(-3)=0=f(3)$

(c) $f(x)=10+5x-x^3-7x^2+x^4$, if $f\left(\sqrt{5}\right)=0$ (d) $f(x)=x^4-5x^2-2x^3+8x+4$, if $f\left(1-\sqrt{2}\right)=0$

8. (**Calculator permitted**) If $P(x)=8x^5-14x^4-22x^3+57x^2-35x+6$, list all possible rational zeros, then find the simplified, exact real zeros. Use the calculator to help you find rational roots, and use the quadratic formula if necessary.

9. Write an equation, in reduced factored form, of a polynomial, *f*, of lowest degree with the following properties: $f(-1) = f(-4) = f\left(-\sqrt{2}\right) = f\left(2+\sqrt{3}\right) = 0$ and $f(0) = -7$.

10. Find the remainder of each of the following, then decide if the divisor is a factor of the dividend.

(a) when $9x^{2222} - 12x^{1946} + 33x^{565} + 26x$ is divided by $x+1$

(b) when $47x^{5769} - 3x^{400} + 735$ is divided by x

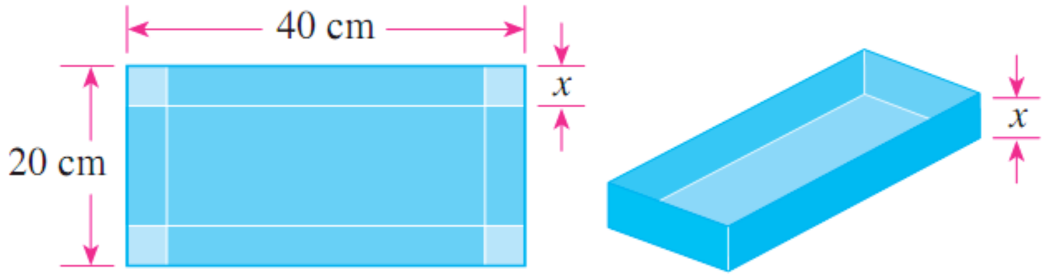
11. Find the given value of *k* so that the given divisor $D(x)$ is a factor of $P(x)$.

(a) $P(x)=3x^4+kx^2-2x+1,\ D(x)=x+1$

(b) $P(x)=2x^3+kx^2+kx-5,\ D(x)=x-2$

(c) $P(x)=3x^{1000}+kx^{48}-2x^{15}+kx,\ D(x)=x-1$

12. (Calculator permitted) An open-top box is to be made by taking a piece of cardboard 20 cm by 40 cm, cutting squares of side length x cm from each corner, and folding up the sides.



(a) Write an equation (in expanded form) of a polynomial function $V(x)$, representing the volume of the box (in cm^3) in terms of x.

(b) What is the domain of $V(x)$? For what values of x is $V(x) > 0$? What is the relevant domain, that is, what values of x make sense in the context of this problem?

(c) If the volume of the box is to be $1500\,\text{cm}^3$, show that the box can be constructed in two different ways, and find the exact dimensions of the box in each case.

(d) What dimensions will give the maximum volume of the box?