2.1-2.5 Xtra Practice Probs

① $f(x)=x^2+3$, $g(x)=2\sqrt{4-2x}$, $m(x)=\frac{5}{x+3}$, $n(x)=3x+7$, $p(x)=5-2x$

(a) $h(x)=g(n(x))$
$=2\sqrt{4-2(3x+7)}$

Domain: $4-2(3x+7)\geq 0$
$4-6x-14\geq 0$
$-6x\geq 10$
$x\leq -\frac{5}{3}$

$\boxed{\text{so } D_h: \{x \mid x\leq -\frac{5}{3}\}}$

Simplify:
$h(x)=2\sqrt{4-6x-14}$
$\boxed{h(x)=2\sqrt{-10-6x}}$

(b) $h(x)=f(g(x))$
$=(2\sqrt{4-2x})^2+3$

Domain: $4-2x\geq 0$
$-2x\geq -4$
$x\leq 2$
$\boxed{\text{So } D_h: \{x \mid x\leq 2\}}$

Simplify: $h(x)=(2\sqrt{4-2x})^2+3$
$h(x)=2^2\cdot(\sqrt{4-2x})^2+3$
$h(x)=4(4-2x)+3$
$h(x)=16-8x+3$
$\boxed{h(x)=19-8x}$

(c) $h(x)=m(m(x))$
$=\dfrac{5}{\frac{5}{x+3}+3}$

Domain: $x+3\neq 0$, $\boxed{x\neq -3}$
$\frac{5}{x+3}+3\neq 0$
$\frac{5}{x+3}\neq -3$
$x+3\neq -\frac{5}{3}$
$x\neq -\frac{5}{3}-3$
$x\neq \frac{-14}{3}$

$\boxed{\text{So } D_h: \{x \mid x\neq -3, -\frac{14}{3}\}}$

Simplify: $h(x)=\dfrac{5}{\frac{5}{x+3}+3}\left(\dfrac{x+3}{x+3}\right)$
$h(x)=\dfrac{5(x+3)}{5+3(x+3)}$
$h(x)=\dfrac{5x+15}{5+3x+9}$
$\boxed{h(x)=\dfrac{5x+15}{3x+14}}$

(d) $h(x)=\dfrac{g(x)}{n(p(x))}$
$=\dfrac{2\sqrt{4-2x}}{3(5-2x)+7}$

Domain: $4-2x\geq 0$
$-2x\geq -4$
$\boxed{x\leq 2}$

$3(5-2x)+7\neq 0$
$15-6x+7\neq 0$
$-6x\neq -22$
$\boxed{x\neq \frac{11}{3}}$
but $\frac{11}{3}>2$

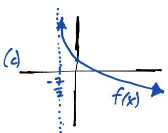
$\boxed{\text{So } D_h: \{x \mid x\leq 2\}}$

Simplify: $h(x)=\dfrac{2\sqrt{4-2x}}{3(5-2x)+7}$
$h(x)=\dfrac{2\sqrt{4-2x}}{15-6x+7}$
$h(x)=\dfrac{2\sqrt{4-2x}}{22-6x}$
$h(x)=\dfrac{\cancel{2}\sqrt{4-2x}}{\cancel{2}(11-3x)}$
$\boxed{h(x)=\dfrac{\sqrt{4-2x}}{11-3x}}$

② $f(x)=\frac{2}{5}-\frac{1}{5}\ln(1+\frac{2}{7}x)+\frac{2}{3}$

(a) $f(x)=-\frac{1}{5}\ln(\frac{2}{7}x+1)+\frac{2}{3}+\frac{2}{5}$
$\boxed{f(x)=-\frac{1}{5}\ln(\frac{2}{7}(x+\frac{7}{2}))+\frac{16}{15}}$

(b) $f(0)=-\frac{1}{5}\ln(\frac{2}{7}(0+\frac{7}{2}))+\frac{16}{15}$
$f(0)=-\frac{1}{5}\ln 1+\frac{16}{15}$ $\boxed{*\ln 1=0}$
$\boxed{f(0)=\frac{16}{15}}$

(c)


(d) $\boxed{D_f: \{x \mid x>-\frac{7}{2}\}}$ * right of VA

(e) $\boxed{R_f: \mathbb{R}}$

(f) $\boxed{\text{VA at } x=-\frac{7}{2}}$

(g) $\boxed{\lim_{x\to\infty} f(x)=-\infty}$

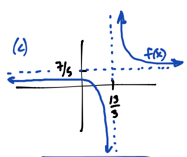
(h) $\boxed{\lim_{x\to-\infty} f(x)=\text{DNE}}$

*graph does not live left of VA.

③ $f(x)=\frac{8}{3x-13}+\frac{7}{5}$

(a) $\boxed{f(x)=8\left(\frac{1}{3(x-\frac{13}{3})}\right)+\frac{7}{5}}$

(b) $f(0)=\frac{8}{-13}+\frac{7}{5}$
$f(0)=\frac{40-91}{-65}$
$\boxed{f(0)=\frac{51}{65}}$

(c)


(d) $\boxed{D_f: \{x \mid x\neq \frac{13}{3}\}}$

(e) $\boxed{R_f: \{y \mid y\neq 7/5\}}$

(f) $\boxed{\text{HA @ } y=7/5}$
$\boxed{\text{VA @ } x=\frac{13}{3}}$

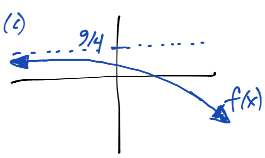
(g) $\boxed{\lim_{x\to\infty} f(x)=\frac{7}{5}}$

(h) $\boxed{\lim_{x\to-\infty} f(x)=\frac{7}{5}}$

④ $f(x)=\frac{9}{4}-\frac{1}{5}e^{(\frac{2}{5}x+\frac{2}{3})}$

(a) $\boxed{f(x)=-\frac{1}{5}e^{(\frac{2}{5}(x+\frac{5}{3}))}+\frac{9}{4}}$

(b) $f(0)=-\frac{1}{5}e^{2/3}+\frac{9}{4}$
$\boxed{f(0)=\frac{9}{4}-\frac{e^{2/3}}{5}>0}$

(c)


(d) $\boxed{D_f: \mathbb{R}}$

(e) $\boxed{R_f: \{y \mid y<\frac{9}{4}\}}$ *Below HA

(f) $\boxed{\text{HA at } y=\frac{9}{4}}$

(g) $\boxed{\lim_{x\to\infty} f(x)=-\infty}$

(h) $\boxed{\lim_{x\to-\infty} f(x)=\frac{9}{4}}$