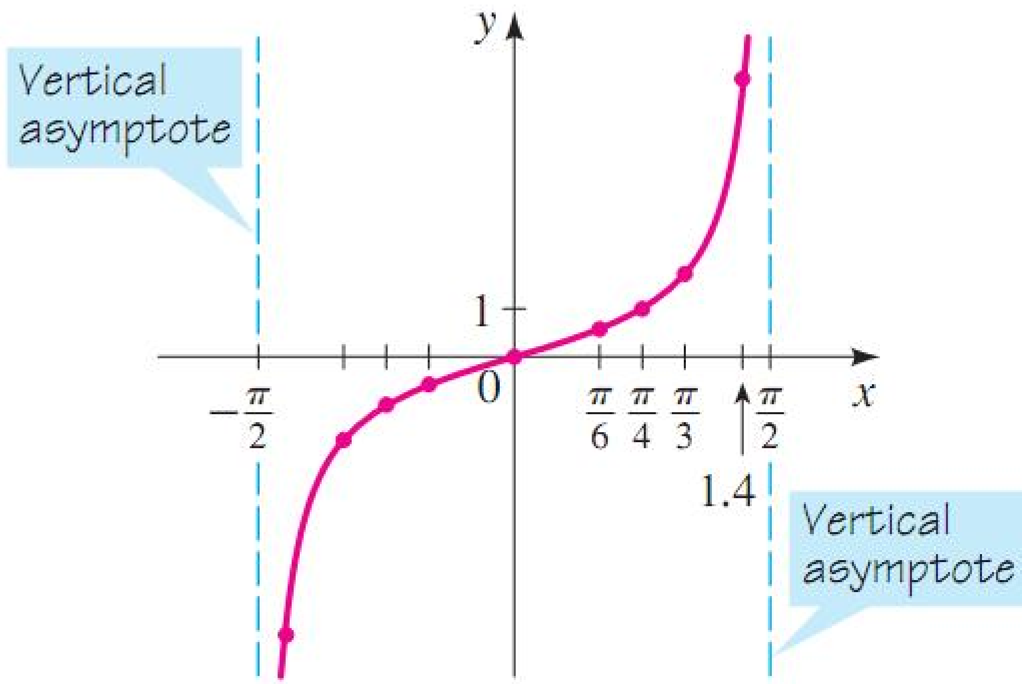
Chapter 5.6: The Other Trig Functions

The other four trig functions, tangent, cotangent, cosecant, and secant are not sinusoids, although they are still periodic functions. Each of the graphs of these functions have discontinuities.

We'll start with the graph of $y = \tan x$.

If we were to create a table of values and plot points, we would see that $\tan x = \tan(x + \pi)$ for all x in the domain of tangent. This means that the period of $y = \tan x$ is only π, rather than 2π. The graph is undefined for all odd $\frac{\pi}{2}$. The principal branch of $y = \tan x$ is shown below.



Example 1:
List all the properties of the function $y = \tan x$, including domain, range, increasing/decreasing behavior, equations of vertical asymptotes, zeros, symmetry, and periodicity.

Example 2:
Sketch several cycles of $y = \tan x$. Since the function $y = \cot x$ is the reciprocal of $y = \tan x$, sketch the graph of $y = \cot x$ by plotting the reciprocal of the function values of $y = \tan x$. Then, list all the properties of the function $y = \cot x$, including domain, range, increasing/decreasing behavior, equations of vertical asymptotes, zeros, symmetry, and periodicity. Write an equation for the graph of $y = \cot x$ in terms of tangent.

Example 3:
Sketch several cycles of the graphs of the following equations. Then, for each, write another equivalent equation of your sketch in terms of the cofunction.

(a) $y = 2\tan x + 1$ (b) $y = -\cot x - 1$ (c) $y = \tan 2\left(x - \frac{\pi}{4}\right)$ (d) $y = 2\cot\left(3x - \frac{\pi}{2}\right)$

The graphs of cosecant and secant are much easier to graph, simply because they are the reciprocal graphs of sine and cosine, respectively.

- The *x*-values of the inflection points on the sinusoidal axis become the vertical asymptotes of the reciprocal graphs.
- The Highs stay high, then "flip" up towards the vertical asymptotes.
- The Lows stay low, then "flip" down towards the vertical asymptotes.

Example 4:
Sketch two cycles of each of the following using the techniques of graphing reciprocals of functions. Then list all the properties of each of the functions, including domain, range, increasing/decreasing behavior, equations of vertical asymptotes, zeros, symmetry, and periodicity.

(a) $y = \csc x$

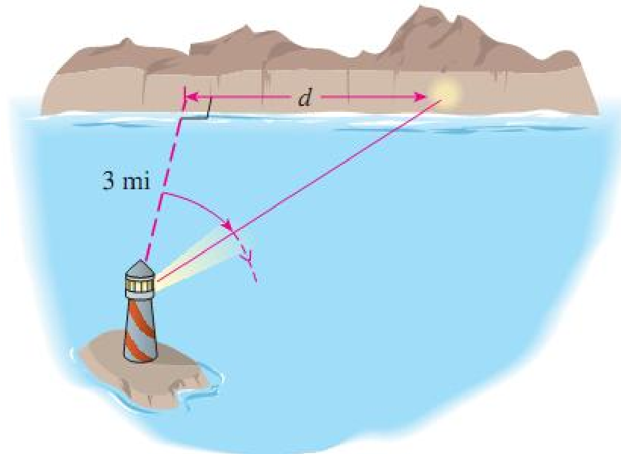
(b) $y = \sec x$

Example 5:
Sketch at least two cycles of the graphs of the following equations. Then, for each, write another equivalent equation of your sketch in terms of the cofunction. Use symmetry to help you simplify the equations before sketching.

(a) $y = \frac{1}{2}\csc(-2x) - 1$

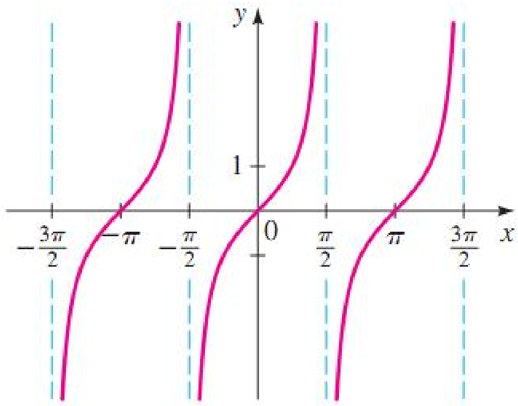
(b) $y = 2\sec\left(-\frac{\pi}{2}x + \pi\right) + 1$

Example 6:
A rotating light on the top of a lighthouse 3 miles out from a straight shore sends out light beams in opposite directions. As the beacon rotates through an angle θ, a spot of light moves along the shore. Let L be the length of the light beam from the lighthouse to spot on the shore. Let d be the directed distance along the shore to the spot light, that is d is positive to the right if you're at the lighthouse facing the shore and negative to the left. Assuming the light makes a full rotation every 2 minutes.

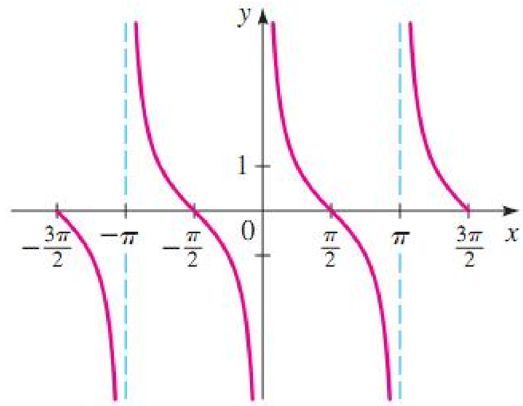


(a) Sketch a graph of the function d for $0 \le t < 2$.
(b) Write an equation for the distance d in miles of the light beam along the shore at time t minutes.
(c) Find $d\left(\frac{1}{6}\right)$, $d\left(\frac{1}{4}\right)$, and $d\left(\frac{1}{3}\right)$.
(d) What is the first positive time, in minutes, that the light beam will be 3000 miles long?
(e) Find $d\left(\frac{5}{6}\right)$. Explain the meaning of this answer.

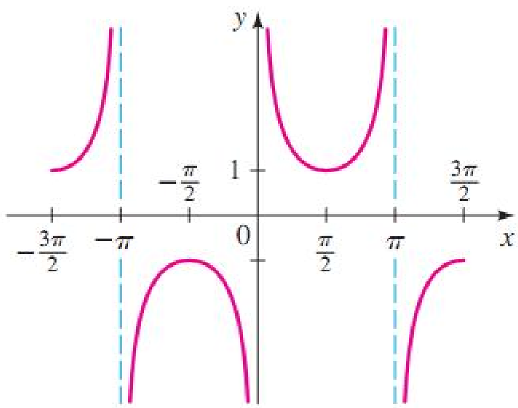
(c) Explain the significance of the asymptote in the graph on either side of $t = \frac{1}{2}$ minutes and $t = \frac{3}{2}$ minutes?



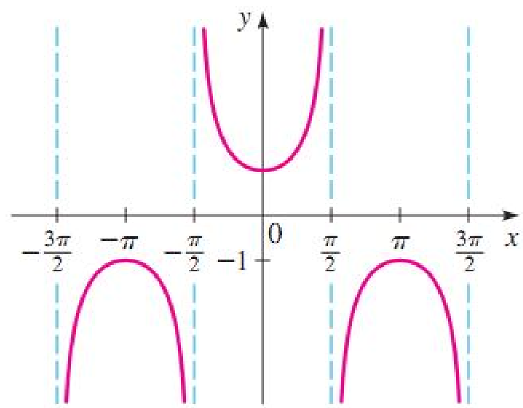
$y = \tan x$



$y = \cot x$



$y = \csc x$



$y = \sec x$

Summary:

- tangent and cotangent have a default period of π. $P = \frac{\pi}{|B|}$
- tangent and cotangent have critical values every HALF a period.
- tangent and cotangent have vertical asymptotes every FULL period.
- tangent starts at an inflection value on the *y*-axis and a VA at the first critical value at $x = \frac{P}{2}$.
- cotangent starts with a VA on the *y*-axis.

- secant and cosecant have a default period of 2π. $P = \frac{2\pi}{|B|}$
- secant and cosecant have critical values every QUARTER of a period.
- secant and cosecant have vertical asymptotes every HALF period.
- secant has an extrema on the *y*-axis and a VA at the first critical value at $x = \frac{P}{4}$.
- cosecant has VA on the *y*-axis.

Example 7:
For each of the following, find, without graphing, find (i) the period, (ii) the range, and (ii) the domain!

(a) $f\left(x\right)=3\tan\left(6-7\pi x\right)+6$

(b) $f\left(x\right)=3\cot\left(6-7\pi x\right)+6$

(c) $g\left(x\right)=6-4\csc\left(\frac{4\pi}{7}x-11\right)$

(d) $g\left(x\right)=6-4\sec\left(\frac{4\pi}{7}x-11\right)$