Chapter 4.5: Exponential and Log Equations

An **exponential equation** is a **conditional equation** in which the variable for which we'd like solve occurs in the exponent. For example,

$$3^x = 11$$

For such an equation, the difficulty is in getting the x "down" out of the exponent. Using logs takes all the difficulty out of it.

Example 1:
Solve for x: $3^x = 11$. Think of a reasonable answer first. Get a simplified exact answer, then give a three decimal approximation.

The method used in Example 1 is typical of how we will solve exponential equations.

In some cases, we can solve exponential equations without having to use logs, but only the fact that exponential functions are one-to-one

Solving by the GTBTS method (Get The Bases The Same method)

If $b^x = b^y$, where $b > 0,\ b \neq 1$, then $x = y$

Example 2:
Solve each of the following by the GTBTS method.

(a) $9^{2x} = 27^{x-1}$ (b) $100\left(\frac{1}{5}\right)^{x/4} = 4$ (c) $\left(\frac{1}{4}\right)\left(\sqrt[3]{16}\right)^{x+4} = \left(\frac{\sqrt[5]{8}}{2}\right)^{2x-1}$

Example 3:
Solve the following equations. Give a simplified, exact answer and a three-decimal approximation. Verify graphically with your calculator.

(a) $9e^{2x} = 30$ (b) $7^{x-5} = 2^{3x+4}$ (c) $3\cdot 5^{2x-1} = 4\cdot 6^{1+x}$

Of course, equations can get interesting.

Example 4:
Solve the following equations algebraically.

(a) $\dfrac{e^x}{e^x - 1} = 5$ (b) $3xe^x = x^2 e^x$ (c) $3e^{-x} - e^x = -2$ (d) $\dfrac{500}{1+25e^{0.3x}} = 200$

A logarithmic equation is a conditional equation in which a log of a variable occurs (a variable occurs in the argument). For example.

$$\log_3\left(x+2\right) = 4$$

It is easier to solve such equations by converting them to their equivalent exponential form, then using the techniques already learned to solve the equation. **Since logs have restricted domains, it's important to find the domain of the solution set prior to solving a log equation.**

Example 5:
Solve $\log_3\left(x+2\right) = 4$ two different ways, by (a) converting it to exponential and by (b) exponentiating both sides.

Example 6:
Solve $\log x^2 = 2$ two different ways, by (a) using the properties of logs to simplify the log expression and by (b) converting it directly exponential. Are you solution sets the same? Which is correct?

Example 7:
Solve the following log equations.
(a) $4+3\log_3(2x)=16$ (b) $\log(x+2)+\log(x-1)=1$ (c) $2\log_4(x-5)-3\log_4(x-5)=1$

(d) $\ln(3x-2)+\ln(x-1)=2\ln x$ (e) $\log_2 x+\log_2 3x^2=\log_2 24$ (f) $\log_{100} x^2-\log x^3=6$

(g) $\log_{0.1}(x-2)+\log x=1$ (h) $\log_2(3x+3)=4+\log_4 x$ (i) $\log(x-5)+\log(-x-3)=4$

Example 8:
Using a calculator, solve the following equations or inequalities.
(a) $e^x = x^2$ (b) $\ln x > \sqrt[3]{x}$ (c) $x^2 \le 2\ln(x+2)$

Example 9:
Find the *x*- and *y*-intercepts of the following, then sketch the graphs.
(a) $y = \ln(3-x)+7$ (b) $y = 4\cdot 7^x - 3$

Example 10:
Find the inverse of the following functions.
(a) $f(x) = 5^{x-4}+3$ (b) $g(x) = \log_5(6x+4)-3$

Example 11:
Solve the following **literal equations** for the indicated variable.
(a) $T = T_s + D_0 e^{-kt}$ for t. (b) $y = \dfrac{a}{1+be^{-(x-c)/d}}$ for x