Chapter 4.4: Properties of Logs

Remember that logs are exponents. Consequently, the properties of logs are similar to those of exponents.

Property I

$$x^a \cdot x^b = x^{a+b} \text{ so } \ln(a \cdot b) = \ln a + \ln b$$

Example 1:

Expand: (a) $\ln(3 \cdot 5)$ (b) $\ln 4xy$

Condense: (c) $\ln 4x + \ln 3x + \ln 2$

Property II

$$\frac{x^a}{x^b} = x^{a-b} \text{ so } \ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

Example 2:

Expand: (a) $\ln\left(\frac{4}{3}\right)$ (b) $\ln\left(\frac{7}{6x}\right)$

Condense: (c) $-\ln 3 - \ln x - \ln 5x$

Property III

$$\left(x^a\right)^b = x^{a \cdot b} \text{ so } \ln a^b = b \cdot \ln a$$

Example 3:

Expand: (a) $\ln 2^3$ (b) $\ln x^2$

Condense: (c) $2\ln x + 2\ln 3 - 4\ln y$ (d) $e^{2\ln x}$

Example 4:
Prove Property II and Property III using the other properties of logs.

Example 5:

Expand in one fell swoop: $\log_7\left(\dfrac{5x^2\sqrt[3]{(2y-1)^2}}{2y\sqrt{x+1}}\right)$

Example 6:

Condense in multiple fell swoops, then simplify: $-3\ln 2-\dfrac{1}{2}\ln x+2\ln(3x)+\ln 2y^2$

Example 7:
Use the properties of logs to evaluate the following:
(a) $\log_4 2+\log_4 32$ (b) $\log_2 80-\log_2 5$

AVOID THESE COMMON ERRORS WHEN WORKING WITH LOGS

$$\log_b(x+y)\neq\log_b x+\log_b y \qquad \frac{\log_b x}{\log_b y}\neq\log_b\left(\frac{x}{y}\right)$$

$$(\log_b x)^a\neq a\log_b x \qquad \log_b ax^n\neq n\log_b ax$$

Example 8:
If $\ln 2 \approx 0.7$, evaluate the following:

(a) $\ln 8$ (b) $\ln\left(\frac{1}{4}\right)$ (c) $\ln\sqrt{2}$ (d) $\log_4 \sqrt{8}$

Log functions are used to model a variety of situations involving human behavior. One such behavior is how quickly we forget things we have learned. For example, if you learn math at a certain performance level, (say 90% on a test) and then don't use those math skills for a while, how much will you retain after a week, a month, or a year? Hermann Ebbinghaus (1850-1909) studied this phenomenon and formulated logarithmic law to model this.

Example 9:
Ebbinghaus' Law of Forgetting states that if a task is leaned at a performance level P_0, then after a time interval t the performance level P satisfies

$$\log P = \log P_0 - c\log(t+1)$$

Where c is a constant that depends on the type of task and t is measured in months.
(a) Solve for *P*.
(b) If your score on a math test is a 90, what score would you expect to get on a similar test after two months? After a year? (Assume $c = 0.2$)

Example 10:
We can use the change of base formula to write equivalent equations of the same logarithmic graph. Describe how the graph of $f(x) = \log_{1/2} x$ can be transformed to become the graph of $g(x) = \log_6 x$. Verify on the graphing calculator.