Chapter 3.3: The Intermediate Value Theorem

Functions like polynomials that are smooth and continuous over intervals are very special functions. Such functions have special properties and consequences of their continuity stated as theorems.

One such theorem is called the **Intermediate Value Theorem**, or **IVT** for short.

IVT

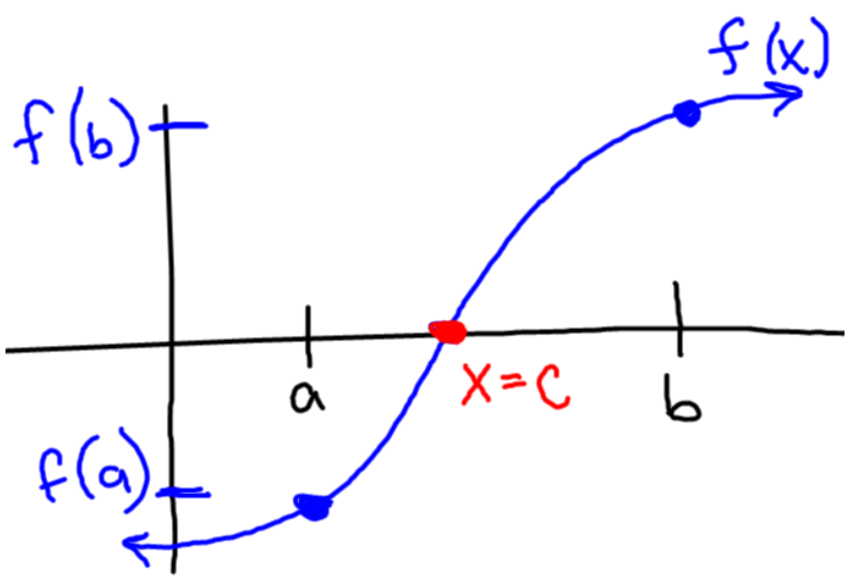
If a function $f(x)$ is continuous on a closed interval $[a,b]$, then there exists a $c \in [a,b]$ such that $f(a) \le f(c) \le f(b)$ or $f(b) \le f(c) \le f(a)$.

Right now, we are going to use the IVT specifically for polynomials specifically to prove the existence of zeros on an interval. Here's a nicer version that will suit our needs:

IVT for Polynomial Zeros

If $f(x)$ is a polynomial function and $f(a)$ and $f(b)$ have different signs, then there exists at least one zero/root between $x = a$ and $x = b$

This is very intuitive if you think about it: the only way to go from positive to negative or negative to positive values through a continuum is by going through ZERO!



A direct consequence of this theorem is that between any two successive polynomial zeros, the graph of the function lies **entirely above** or **entirely below** the *x*-axis.

This theorem is an example of an existence theorem—it tells us only that a zero exists, but it doesn't tell us where it is or how to find it. Knowing that something exists, though, is a great detail to know prior to going out and looking for it!!

Example 1:
Prove that an x-intercept exists for the function $f(x) = 2x^3 - 3x^2 + x - 1$ on the interval $[-1, 2]$

Example 2:
Using your graphing calculator, numerically find the x-intercept for $f(x) = 2x^3 - 3x^2 + x - 1$ on $[-1, 2]$ to three decimal places. Verify your answer graphically. Start you TblStart at -1 and set your ΔTbl to 1.

Example 3:
Using the IVT, prove that for the function $f(x) = -2\sqrt{x+5} + 3$, there exists an $x = c$, $c \in (-1, 11)$, such that $f(c) = -4$. Find this value graphically and algebraically to confirm.