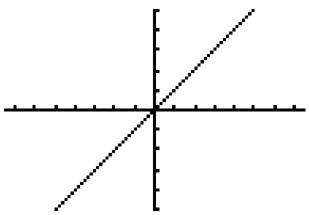
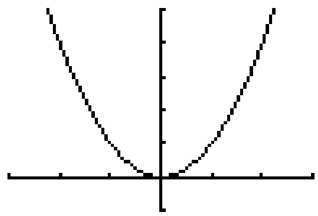
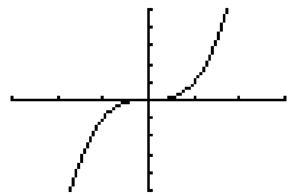
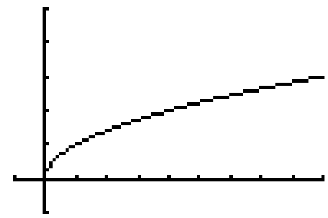
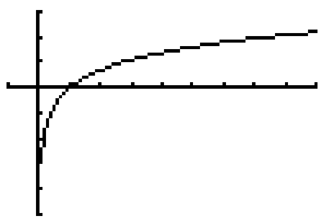
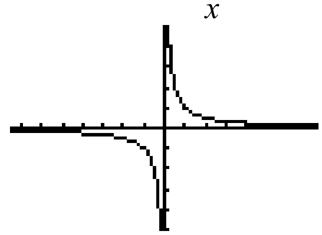
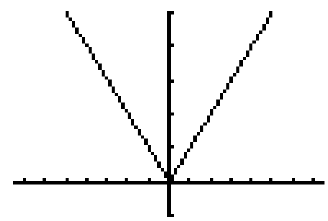
Chapter 2.4: Parent Functions & Transformations

In Algebra II, you had experience with basic functions like linear, quadratic, and hopefully a few others. Additionally, you learned how to **transform** these **basic parent functions** using a sequence of reflections, dilations, and/or translations. In this section, we will quickly review these parent functions and transformations as well as learn a few new ones.

Memorize the following graphs, their equations, and all information from the last two sections about each of them.

$f(x)=x$  Linear Function	$f(x)=x^2$  Quadratic Function	$f(x)=x^3$  Cubic Function	$f(x)=\sqrt{x}$  Square Root Function
$f(x)=e^x$  Exponential Function	$f(x)=\ln x$  Natural Log Function	$f(x)=\dfrac{1}{x}$  Reciprocal Function	$f(x)=\lvert x\rvert$  Absolute Value Function

Example 1:
Sketch the following piecewise function. Check for the domain and range, continuity, relative extrema, and symmetry.

$$g(x)=\begin{cases} x, & x\le -1 \\ x^2, & -1<x<0 \\ \sqrt{x}, & 0\le x<1 \\ x^{-1}, & x>1 \end{cases}$$

Now that we are comfortable with these eight parent functions, lets play with them a little be by changing a thing or two in their equations and seeing how it affects the graph: Let's transform them.

Follow these easy steps to easily sketch transformations of any parent function.

1. Identify the parent or original function.
2. Make sure the equation of the function is in **Standard Transformation Form**.
3. Sketch the function using your Dear Aunt Sally's rules, showing where important features of the new, transformed graph.

Standard Transformation Form

A function $g(x)$ is said to be written in Standard Transformation Form if it is written as

$$g(x) = Af\left(B(x-C)\right) + D$$

where $f(x)$ is the parent or original function.

There are two dynamics in this form that you must thoroughly understand: (1) **Multiplication versus Addition**, and (2) **Inside verses Outside** the parent function.

(1) **Multiplication versus Addition:** *A* and *B* are being multiplied while *C* and *D* are being added (or subtracted).

- Multiplying changes the SHAPE or ORIENTATION of the graph. Such transformations are called **Dilations** or **Reflections** (respectively).
- Adding changes the LOCATION of the graph. Such transformations are called **Translations** (or **Shifts**).
- Like the rules of your Dear Aunt Sally, when sketching a new graph by a sequence of successive transformations, multiplication transformations must be done before addition transformations.
- If you understand the sequence of transformations, you should be able to sketch transformations in a single step without going through the transformations.

(2) **Inside verses Outside:** *B* and *C* are on the inside of the parent function while *A* and *D* are on the outside of the function.

- Changes to the **inside** of the function affect the *x*-values, or the function's inputs. This means the **horizontal nature** of the graph will change, either dilated, reflected, or shifted.
- Changes to the **outside** of the function will affect the *y*-values, or the function's outputs. This means the **vertical nature** of the graph will change, either dilated, reflected, or shifted.
- ***As a rule of thumb, inside changes are OPPOSITE of what they immediately appear to be, while outside changes are EXACTLY as you would suspect. We will see this soon in an example, and if you'd like, I can even show you why.

Example 3:
For the following function, identify the parent function, write the equation in standard transformation form, then identify the values of *A, B, C,* and *D*. $h(x)=3+\frac{1}{2}\sqrt{2-4x}$.

Example 4:
Write the equations of the functions if the same transformations applied above are applied to the following parent functions:

(a) $y=f(x)$

(b) $f(x)=x^2$

(c) $f(x)=\frac{1}{x}$

(d) $f(x)=x^3$

(e) $f(x)=\ln x$

(f) $f(x)=e^x$

As you work through more and more examples, the shift transformations will become very intuitive. Dilations, however, can be tricky to interpret and tricky to graph, especially since several algebra texts do a poor job of describing what these transformations actually do. I will teach you what I expect you to do.

NOTES:
- We always interpret a dilation as a stretch (if it gets bigger) or a compression (if it gets smaller).
- We can abbreviate "by a factor of" as "bfo."
- When we say "bfo," the number that follows MUST BE GREATER THAN ONE!!

- $|A|$ is the vertical dilation factor
  - If $|A|>1$, ______________________________
  - If $0<|A|<1$, ______________________________
  - If $A<0$, ______________________________
- $|B|$ is the horizontal dilation factor
  - If $|B|>1$, ______________________________
  - If $0<|B|<1$, ______________________________
  - If $B<0$, ______________________________
- C is the horizontal shift constant
  - If $C>0$, ______________________________
  - If $C<0$, ______________________________
- D is the vertical shift constant
  - If $D>0$, ______________________________
  - If $D<0$, ______________________________

Example 5:
For each of the following, put the equation in standard transformation form, identify the parent function, explain the sequence of transformations, then sketch a graph showing the important information.

(a) $f(x)=\dfrac{-3}{2+2x}-1$ (b) $g(x)=3-\dfrac{e^{4x+7}}{5}$ (c) $u(x)=\pi+\ln\left(5-\dfrac{5x}{6}\right)+e$

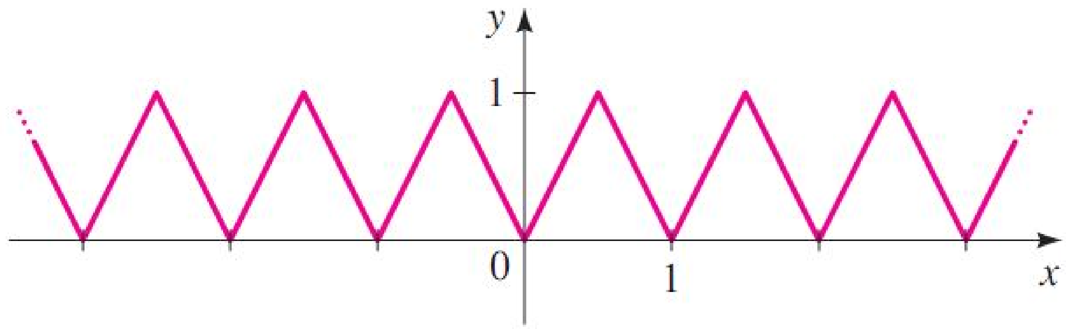
(d) $m(x) = 6 + \left|2 - \frac{x}{2} - 7\right|$ (e) $p(t) = \sqrt{2 - \frac{3}{2}t} + 1$ (f) $x(t) = \frac{4t-3}{2-5t}$

Sometimes (most times) there are equivalent ways to produce the same graph using different methods.

Example 6:
If $y = f(x)$ is a parent function and $g(x) = f(9x)$, discuss two different ways (if possible) to generate the graph of g from f using standard transformations for the following functions:

(a) $f(x) = x$ (b) $f(x) = x^2$ (c) $f(x) = x^3$ (d) $f(x) = \sqrt{x}$

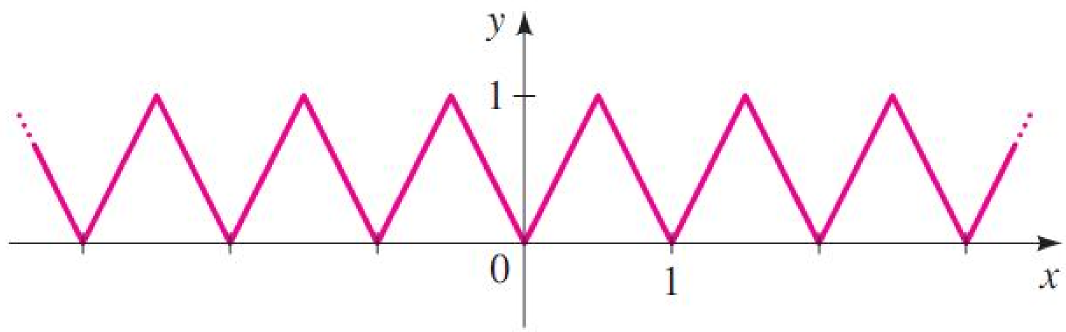
(e) $f(x) = x^{-1}$ (f) $f(x) = |x|$ (g) $f(x) = \ln x$ (h) $f(x) = e^x$



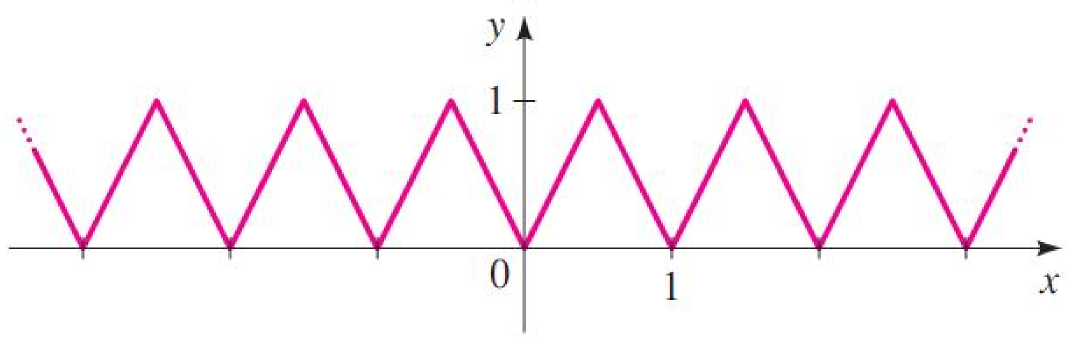
Example 7:

For each of the following, sketch the indicated transformation of the given graph of $y = f(x)$ (ABOVE).

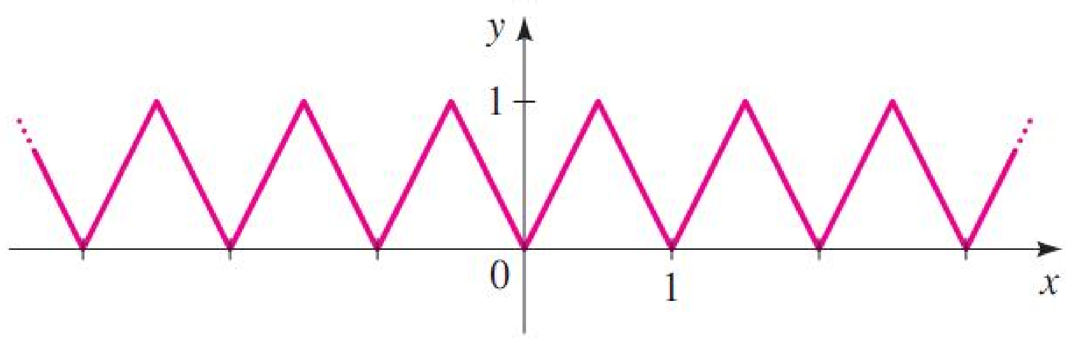
(a) $y = f(3x)$



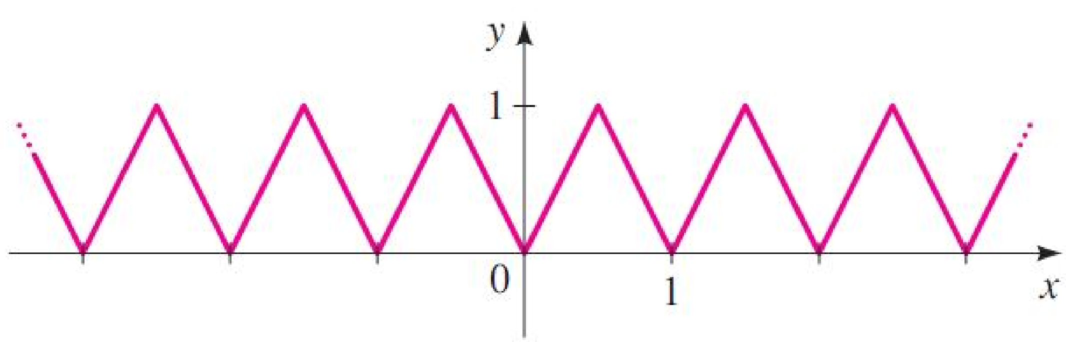
(b) $y = f\left(\frac{x}{3}\right)$



(c) $y = -3f(x)$



(d) $y = \frac{f(-x)}{3}$



Two VERY special transformations (and a bonus third)!—ABSOLUTE VALUES

Example 8:
Using your calculator, sketch the graph of $y = \ln x$ on the window $X[-4,4]$, $Y[-3,3]$. In a different $y =$ and using a different line quality, sketch (a) $y = |\ln x|$ (b) $y = \ln|x|$ and (c) $y = |\ln|x||$. What do you notice? Can you develop some rules of thumb for these transformations? Try them out on a different parent function (like $f(x) = x$)? Would you expect to see visible transformations on all of our parent functions? Why or why not? Would you like this section to finally be over?

(a) $y = |\ln x|$

(b) $y = \ln|x|$

(c) $y = |\ln|x||$.

Example 9:
Use your new rules to sketch the following transformations of the graph of $f(x)$ shown at right

(a) $y = |f(x)|$

(b) $y = f(|x|)$

(c) $y = |f(|x|)|$