Name KEY Date Period

PreAP Precalculus TEST: 6.1-6.4: NO Calculator. Show ALL steps

Answers will vary

Part I: Trig Proofs—Prove 4 out of 5 Identities. Show all steps including substitutions and algebraic procedures.

1. $\csc x - \cos x \cot x = \sin x$

$\frac{1}{\sin x} - \cos x\left(\frac{\cos x}{\sin x}\right)$

$\frac{1-\cos^2 x}{\sin x}$

$\frac{\sin^2 x}{\sin x}$

$\sin x$

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2. $\dfrac{\cos(-x)}{\sec(-x)+\tan(-x)} = 1-\sin(-x)$

$\frac{\cos x}{\sec x - \tan x}$ | $1+\sin x$

$\frac{\cos x}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}}$

$\left(\frac{\cos x}{\cos x}\right)\frac{\cos x}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}}$

$\frac{\cos^2 x}{1-\sin x}$

$\frac{1-\sin^2 x}{1-\sin x}$

$\frac{\cancel{(1-\sin x)}(1+\sin x)}{\cancel{(1-\sin x)}}$

$1+\sin x$

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3. $\dfrac{1+\tan x}{1-\tan x} + \dfrac{1+\cot x}{1-\cot x} = 0$

$\frac{1+\frac{1}{\cot x}}{1-\frac{1}{\cot x}} + \frac{1+\cot x}{1-\cot x}$

$\left(\frac{\cot x}{\cot x}\right)\frac{1+\frac{1}{\cot x}}{1-\frac{1}{\cot x}} + \frac{1+\cot x}{1-\cot x}$

$\left(\frac{-1}{-1}\right)\frac{\cot x+1}{\cot x-1} + \frac{1+\cot x}{1-\cot x}$

$\frac{-\cot x-1}{1-\cot x} + \frac{1+\cot x}{1-\cot x}$

$\frac{-\cot x-1+1+\cot x}{1-\cot x}$

$\frac{0}{1-\cot x}$

0

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4. $\sin^2 x\cos^3 x = \left(\sin^2 x - \sin^4 x\right)(\cos x)$

$\cos x(\sin^2 x\cdot\cos^2 x)$

$\cos x(\sin^2 x(1-\sin^2 x))$

$\cos x(\sin^2 x - \sin^4 x)$

$(\sin^2 x - \sin^4 x)\cos x$

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5. $\dfrac{\sin x}{1+\sec x} = \dfrac{\sin x\sin\left(\frac{\pi}{2}-x\right)}{\cos x+1}$

$\frac{\sin x\cdot\cos x}{\cos x+1}$

$\frac{\sin x\cdot\cos x}{(\cos x+1)}\left(\frac{\sec x}{\sec x}\right)$

$\frac{\sin x\cdot 1}{1+\sec x}$

$\frac{\sin x}{1+\sec x}$

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Part II: Solving Trigonometric Equations— For each of the following, solve each trig function without a calculator, where $0 \le x < 2\pi$. Show all work and give exact answers.

6. $\cos 2x + 5\cos x = 2$

$\cos^2 x - \sin^2 x + 5\cos x - 2 = 0$

$\cos^2 x - (1-\cos^2 x) + 5\cos x - 2 = 0$

$\cos^2 x - 1 + \cos^2 x + 5\cos x - 2 = 0$

$2\cos^2 x + 5\cos x - 3 = 0$

$(2\cos x - 1)(\cos x + 3) = 0$

$\cos x = \frac{1}{2}$ or $\cos x = -3$ No soln

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

7. $-6\sin 3x = 3\sqrt{3}$

$\sin 3x = \frac{3\sqrt{3}}{-6}$

$\sin 3x = -\frac{\sqrt{3}}{2}$

$\begin{cases} 3x = \frac{4\pi}{3} + 2\pi n \\ 3x = \frac{5\pi}{3} + 2\pi n \end{cases}$

$\begin{cases} x = \frac{4\pi}{9} + \frac{2\pi}{3} n \\ x = \frac{5\pi}{9} + \frac{2\pi}{3} n \end{cases}$

$\boxed{x = \frac{4\pi}{9}, \frac{10\pi}{9}, \frac{16\pi}{9}, \frac{5\pi}{9}, \frac{11\pi}{9}, \frac{17\pi}{9}}$

8. $4\tan^2 x = 3\tan^2 x + 3$

$4\tan^2 x - 3\tan^2 x = 3$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

Part III: Trig Proofs

Prove 3 out of 4 Identities. Show all steps including substitutions and algebraic procedures.

9. $\cos^2\left(\frac{-x}{2}\right) = \frac{1+\sec x}{2\sec x}$

$\cos^2\left(\frac{1}{2}x\right)$

$\frac{1}{2}(1+\cos x)$

$\frac{(1+\cos x)}{2}\left(\frac{\sec x}{\sec x}\right)$

$\frac{\sec x + 1}{2\sec x}$

$\frac{1+\sec x}{2\sec x}$

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10. $\sin^5 x = \left(\frac{1}{8}\sin x\right)(3 - 4\cos 2x + \cos 4x)$

$(\sin^2 x)^2 \sin x$

$(1-\cos^2 x)^2 \sin x$

$(1 - 2\cos^2 x + \cos^4 x)\sin x$

$\sin x\left(1 - 2\left[\frac{1}{2}(1+\cos 2x)\right] + \left[\frac{1}{2}(1+\cos 2x)\right]^2\right)$

$\sin x\left(1 - (1+\cos 2x) + \frac{1}{4}(1 + 2\cos 2x + \cos^2 2x)\right)$

$\sin x\left(1 - 1 - \cos 2x + \frac{1}{4} + \frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x\right)$

$\sin x\left(\frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\left[\frac{1}{2}(1+\cos 4x)\right]\right)$

$\sin x\left(\frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{8} + \frac{1}{8}\cos 4x\right)$

$\sin x\left(\frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x\right)$

$\frac{1}{8}\sin x(3 - 4\cos 2x + \cos 4x)$

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11. $\tan\left(x + \frac{\pi}{4}\right) = \frac{1+\tan x}{1-\tan x}$

$\frac{\tan x + \tan\frac{\pi}{4}}{1 - \tan x \tan\frac{\pi}{4}}$

$\frac{\tan x + 1}{1 - \tan x}$

$\frac{1+\tan x}{1-\tan x}$

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12. $\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x + \tan y}{\tan x - \tan y}$

$\dfrac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{\frac{\sin x}{\cos x} - \frac{\sin y}{\cos y}}$

$\dfrac{\left(\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}\right)\cos x\cos y}{\left(\frac{\sin x}{\cos x} - \frac{\sin y}{\cos y}\right)\cos x\cos y}$

$\frac{\sin x\cos y + \sin y\cos x}{\sin x\cos y - \sin y\cos x}$

$\frac{\sin(x+y)}{\sin(x-y)}$

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