Name KEY Date Period

TEST: 5.1-5.6A—NO Calculator Permitted

Part I—Multiple Choice: Put the capital letter of the correct answer in the blank beside the question number.

E 1. What is the period of the following function? $y = 2 - 9\tan\left(\frac{4\pi}{7} + \frac{3\pi}{5}x\right)$

(A) $\frac{7}{2}$ (B) $\frac{10}{3}$ (C) $\frac{4}{7}$ (D) $\frac{7}{4}$ (E) $\frac{5}{3}$

$P = \frac{\pi}{|B|} = \frac{\pi}{\frac{3\pi}{5}} = \frac{\pi}{1} \cdot \frac{5}{3\pi} = \frac{5}{3}$

B 2. The function $y = \cot x$ has the same domain as the function

(A) $\sin x$ (B) $\csc x$ (C) $\tan x$ (D) $\sec x$ (E) $\cos x$

$\cot\theta = \frac{x}{y}$

$\csc\theta = \frac{r}{y}$

Both are undefined when $y=0$ on Unit Circle at $\theta = 0, \pi, 2\pi, \ldots$

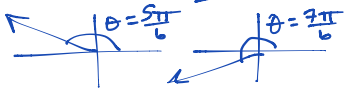
A 3. If $\sec\theta = -\frac{2\sqrt{3}}{3}$, then which of the following angles could θ be?

I. $\frac{5\pi}{6}$ ✓

II. $\frac{4\pi}{3}$ X

III. $\frac{13\pi}{6}$ coterminal with $\frac{\pi}{6}$ X

if $\sec\theta = -\frac{2\sqrt{3}}{3}$ then $\cos\theta = -\frac{\sqrt{3}}{2}$



(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

A 4. Find the domain of $f(x) = -5\csc\left(\frac{3\pi}{4}x - \frac{\pi}{3}\right) + 1$ for $n \in \mathbb{Z}$.

(A) $\left\{x \middle| x \neq \frac{4}{9} + \frac{4}{3}n\right\}$ (B) $\left\{x \middle| x \neq \frac{4}{9} + \frac{2}{3}n\right\}$ (C) $\left\{x \middle| x \neq \frac{\pi}{3} + \frac{4}{3}n\right\}$ (D) $\left\{x \middle| x \neq \frac{2}{3} + 3n\right\}$ (E) $\left\{x \middle| x \neq \frac{10}{9} + \frac{4}{3}n\right\}$

$f(x) = -5\csc\left(\frac{3\pi}{4}\left(x - \frac{4}{9}\right)\right) + 1$

$D_f: \left\{x \middle| x \neq 0 + \frac{4}{9} + \frac{4}{3}n, n \in \mathbb{Z}\right\}$

$x \neq 0 + C + \frac{1}{2}P n$

$\frac{\pi}{3} \div \frac{3\pi}{4}$

$\frac{\pi}{3} \cdot \frac{4}{3\pi}$

$\frac{4}{9}$

$P = \frac{2\pi}{\frac{3\pi}{4}} = \frac{2\pi}{1} \cdot \frac{4}{3\pi} = \frac{8}{3}$

$\frac{1}{2}P = \frac{8}{3} \cdot \frac{1}{2} = \frac{4}{3}$

D 5. What is the range of $y = 7 - 3\sec(6 - 2\pi x)$?

(A) $\{y | 4 \le y \le 10\}$ (B) $\{y | -4 \le y \le 10\}$ (C) $\{y | y \le -4 \text{ or } y \ge 10\}$ (D) $\{y | y \le 4 \text{ or } y \ge 10\}$ (E) all reals

$R: (-\infty, D-|A|] \cup [D+|A|, \infty)$

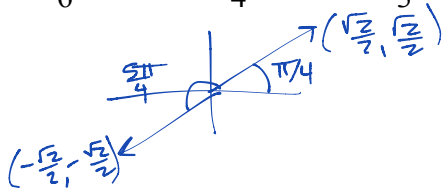
$R: (-\infty, 7-3] \cup [7+3, \infty)$

$R: (-\infty, 4] \cup [10, \infty)$

$R: \{y \mid y \le 4 \text{ or } y \ge 10\}$

B 6. For what angle, $0 \le \theta < 2\pi$, does $\sec\theta = \csc\theta$?

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) No such angle exists



$\sec(\frac{\pi}{4}) = \sqrt{2} = \csc(\frac{\pi}{4})$

$\sec(\frac{5\pi}{4}) = -\sqrt{2} = \csc(\frac{5\pi}{4})$

E 7. If $\sin\pi = A$, $\tan A = B$, then what is $\cot B =$

(A) 0 (B) 1 (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\pi}{4}$ (E) DNE

$\sin\pi = 0$, so $A = 0$ } $\tan A = \tan 0 = 0$, so $B = 0$ } $\cot B = \cot 0 = \text{DNE}$

D 8. For which of the following functions does $f(-x) = f(x)$?

I. $f(x) = \cos x$ ✓

II. $f(x) = \sec x$ ✓

III. $f(x) = \cot x$ ✗

EVEN function with y-axis symmetry

cosx & secx

(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

C 9. Find the domain of $f(x) = 3\tan\left(\frac{\pi}{4} + 4\pi x\right) + 5$ for $n \in \mathbb{Z}$.

(A) $\left\{x \middle| x \ne \frac{1}{16} + \frac{1}{2}n\right\}$ (B) $\left\{x \middle| x \ne \frac{3}{16} + \frac{1}{4}n\right\}$ (C) $\left\{x \middle| x \ne \frac{1}{16} + \frac{1}{4}n\right\}$ (D) $\left\{x \middle| x \ne \frac{3}{16} + \frac{1}{2}n\right\}$ (E) all reals

$f(x) = 3\tan(4\pi x + \frac{\pi}{4}) + 5 \quad P = \frac{\pi}{4\pi} = \frac{1}{4}$

$f(x) = 3\tan(4\pi(x + \frac{1}{16})) + 5 \quad \frac{1}{2}P = \frac{1}{8}$

$D_f: \{x \mid x \ne \frac{1}{8} - \frac{1}{16} + \frac{1}{4}n, n \in \mathbb{Z}\}$

$D_f: \{x \mid x \ne \frac{1}{16} + \frac{1}{4}n, n \in \mathbb{Z}\}$

$x \ne \frac{1}{2}P + C + Pn, n \in \mathbb{Z}$

Part II—Short Answer: Using correct notation, and giving simplified, exact answers, for each of the following,

10. $f(x) = -9 + 2\sec\left(\frac{\pi x}{2} - \pi\right)$

(a) Put $f(x)$ into standard transformation form. ELIMINATE ANY NEGATIVE *B* VALUE BY USING THE SYMMETRY OF THE FUNCTION!!!

$f(x) = 2\sec\left(\frac{\pi}{2}(x-2)\right) - 9$ (✓1)

(b) Find the Period, *P*, of $f(x)$.

$P = \frac{2\pi}{\pi/2} = 4$ (✓2)

(c) Find the Range, *R*, of $f(x)$.

$R_f: (-\infty, -11] \cup [-7, \infty)$ (✓3)

or

$R_f: \{y \mid y \le -11 \text{ or } y \ge -7\}$

(d) Find the Domain, *D*, of $f(x)$.

$f(x) = 2\sec\left(\frac{\pi}{2}(x-2)\right) - 9$

$\frac{1}{4}P = 1$

$\frac{1}{2}P = 2$

$D_f: \{x \mid x \neq 1 + 2 + 2n, n \in \mathbb{Z}\}$

$D_f: \{x \mid x \neq 3 + 2n, n \in \mathbb{Z}\}$ (✓4) (✓5)

11. $g(x)=5\cot\left(\frac{5}{2}-\frac{5\pi}{3}x\right)-8$

(a) Put $g(x)$ into standard transformation form. ELIMINATE ANY NEGATIVE *B* VALUE BY USING THE SYMMETRY OF THE FUNCTION!!!

$g(x)=5\cot\left(-\frac{5\pi}{3}x+\frac{5}{2}\right)-8$

$\frac{5}{2}\div\frac{5\pi}{3}$

$\frac{5}{2}\cdot\frac{3}{5\pi}$

$\frac{3}{2\pi}$

$g(x)=5\cot\left(-\frac{5\pi}{3}\left(x-\frac{3}{2\pi}\right)\right)-8$

$g(x)=-5\cot\left(\frac{5\pi}{3}\left(x-\frac{3}{2\pi}\right)\right)-8$ (√6)

(b) Find the Period, *P*, of $g(x)$.

$P=\frac{\pi}{\frac{5\pi}{3}}=\frac{\pi}{1}\cdot\frac{3}{5\pi}=\frac{3}{5}$ (√7)

(c) Find the Range, *R*, of $g(x)$.

$R_g:\mathbb{R}$ (√8)

or

$(-\infty,\infty)$

or

$\{y\mid y\in\mathbb{R}\}$

(d) Find the Domain, *D*, of $g(x)$.

$g(x)=-5\cot\left(\frac{5\pi}{3}\left(x-\frac{3}{2\pi}\right)\right)-8$

$D_g:\{x\mid x\neq 0+\frac{3}{2\pi}+\frac{3}{5}n, n\in\mathbb{Z}\}$

$D_g:\{x\mid x\neq \underbrace{\frac{3}{2\pi}}_{\text{√9}}+\underbrace{\frac{3}{5}n}_{\text{√10}}, n\in\mathbb{Z}\}$