Name KEY Date_____ Period_____

TEST: 5.1-5.5—Calculator Permitted

Part I: Multiple Choice

A 1. Approximately how many cycles will the function $y=17-3\sin\left(\frac{2\pi}{3}-\frac{7\pi}{5}x\right)$ have between $[0,2\pi)$? = |B|

(A) $\frac{7\pi}{5}$ (B) $\frac{2\pi}{3}$ (C) 3 (D) $\frac{7}{5}$ (E) 7

B=coefficient of x

$|B|=\left|-\frac{7\pi}{5}\right|=\frac{7\pi}{5}$

B or E 2. Which of the following is equal to $f(x)=-A\cos\left(-\frac{\pi}{5}x+\pi\right)+D$?

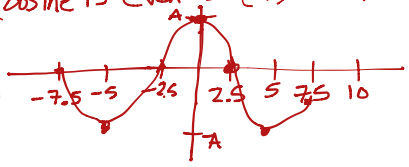
(A) $f(x)=A\sin\left(\frac{\pi}{5}(x+5)\right)+D$ (B) $f(x)=-A\cos\left(\frac{\pi}{5}(x-5)\right)+D$ (C) $f(x)=A\cos\left(\frac{\pi}{5}(x+5)\right)+D$

(D) $f(x)=A\cos\left(\frac{\pi}{5}(x-5)\right)+D$ (E) $f(x)=-A\cos\left(\frac{\pi}{5}(x+5)\right)+D$

$P=\frac{2\pi}{\pi/5}=10$

$f(x)=-A\cos\left(-\frac{\pi}{5}(x-5)\right)+D$

$f(x)=-A\cos\left(\frac{\pi}{5}(x-5)\right)+D$ or $f(x)=-A\cos\left(\frac{\pi}{5}(x+5)\right)+D$ (cosine is even: $\cos(-B)=\cos B$)



A 3. Determine the range of the function $y=\frac{3}{5}-\frac{5}{2}\cos b(x+7)$, where $b>0$.

(A) $\left\{y\middle|-\frac{19}{10}\le y\le\frac{31}{10}\right\}$ (B) $\left\{y\middle|-\frac{31}{10}\le y\le-\frac{19}{10}\right\}$ (C) $\{y|-7\le y\le 7\}$

(D) $\left\{y\middle|-\frac{8}{7}\le y\le-\frac{2}{7}\right\}$ (E) $\left\{y\middle|-\frac{3}{5}\le y\le\frac{3}{5}\right\}$

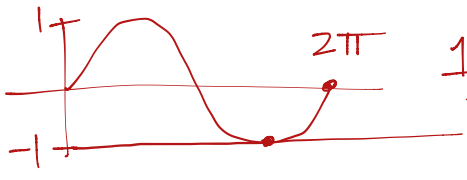
$R:[D-|A|,D+|A|]$, $D=\frac{3}{5}$, $|A|=\frac{5}{2}$

$R:\left[\frac{3}{5}-\frac{5}{2},\frac{3}{5}+\frac{5}{2}\right]$

$R:\left[-\frac{19}{10},\frac{31}{10}\right]$

A 4. The number of solutions to the equation $\sin(45x)=-1$ in the interval $[0,2\pi]$ is

(A) 45 (B) 90 (C) 22.5 (D) 0 (E) $\frac{2\pi}{45}$



1 solution per cycle

sin(45x) has 45 cycles in 2π,

So, there are 45·1=45 solutions

E 5. Which of the following angles is coterminal with $\frac{-5254\pi}{7}$?

(A) $\frac{3\pi}{7}$ (B) $\frac{2\pi}{7}$ (C) $\frac{4\pi}{7}$ (D) $\frac{5\pi}{7}$ (E) $\frac{10\pi}{7}$

$-5254/14 = -375.285\ldots$ rotations

$+375$

$-0.285\ldots$ rot

$\times 14$

-4

1 rotation

So, $\frac{-5254\pi}{7} \rightarrow \frac{-4\pi}{7} \rightarrow -\frac{4\pi}{7} + \frac{14\pi}{7} = \frac{10\pi}{7}$

A 6. What is the period of the following sinusoid: $y = 7 - 18.2\sin\left(\frac{7\pi}{9} + \frac{14}{3}x\right)$?

(A) $\frac{3\pi}{7}$ (B) $\frac{7\pi}{9}$ (C) $\frac{9}{14\pi}$ (D) $\frac{18}{7}$ (E) $\frac{18}{5}$

B = coefficient of x term

$B = \frac{14}{3} = |B|$

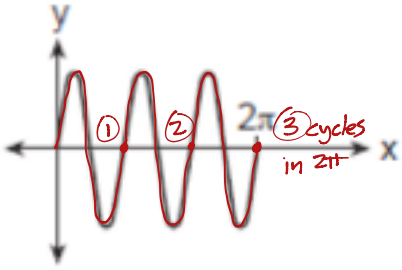
$P = \frac{2\pi}{|B|}$

So, $P = \frac{2\pi}{14/3}$

$P = \frac{2\pi}{1} \cdot \frac{3}{14}$

$P = \frac{3\pi}{7}$

D 7. The function f $f(x) = A\cos(B(x-C)) + D$ is shown at right. What is the value of B in the equation for $f(x)$?



(A) 2π (B) $\frac{2\pi}{3}$ (C) $\frac{3\pi}{2}$ (D) 3 (E) 12π

B = Number of cycles in 2π = 3 (see right)

or

$B = \frac{2\pi}{P}$, $P = \frac{2\pi}{3}$, $B = \frac{2\pi}{2\pi/3} = \frac{2\pi}{1} \cdot \frac{3}{2\pi} = 3$

D 8. A precalculus student drives to school, a tiny stone becomes wedged between the treads in his tire. The height, h in inches, above the ground of the stone can be modeled by the function, $h(t) = 8\cos\left(\frac{\pi t}{50}\right) + 8$, where t is the time in seconds. How fast does the stone rotate in radians per second?

$B = \frac{\pi}{50}$, so $P = \frac{2\pi}{\pi/50} = \frac{2\pi}{1} \cdot \frac{50}{\pi} = 100$ seconds per 1 cycle/rotation

So, $\omega =$ 1 cycle(rotation) per 100 sec $= \frac{1}{P}$

(A) 100 rads/sec (B) 50π rads/sec (C) 25 rads/sec (D) $\frac{\pi}{50}$ rads/sec (E) 100π rads/sec

ω = angular velocity $= \frac{\theta}{t} = \frac{1\text{ rot}}{100\text{ sec}} = \frac{2\pi\text{ rad}}{100\text{ sec}} = \frac{\pi\text{ rad}}{50\text{ sec}} = \frac{\pi}{50}$ rad/sec = B!!

Part II: Free Response

Show all work below the line. Give simplified, exact answers when specified, otherwise report **three decimals**. Avoid intermediate rounding error. Box your final answers, **with units** when appropriate.

9. **Submarine Problem:** You are aboard a submarine submerged in the Pacific Ocean. You make contact with an enemy destroyer. Immediately you start "porpoising" (going deeper and shallower sinusoidally). After 4 minutes you are at your deepest, $d = -1000$ meters and at 9 minutes you reach your next shallowest point at $d = -200$ meters.

(a) Sketch a graph of the depth of the submarine d in meters as a function of time t in minutes.
Sketch two full cycles, labeling your critical values and axes.
Be sure to include 1 critical value to the left of the y-axis.

Sacrificial t-axis

H	A	L	A	H
-1	1.5	4	6.5	9

$\frac{1}{2}P = 9 - 4 = 5$ min

So, $P = 5(2) = 10$ min

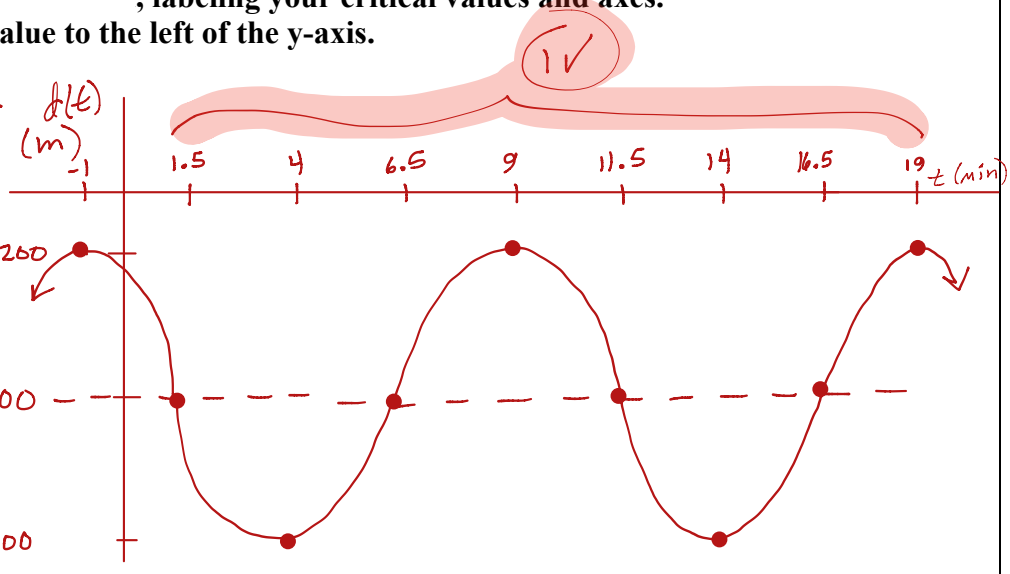
& $\frac{1}{4}P = 2.5$ min

$D = \frac{-200 - 1000}{2}$

$D = -600$

$|A| = 400$

$|B| = \frac{2\pi}{10} = \frac{\pi}{5}$



√1 √2

(b) Write an equation in standard transformation form for the depth $d(t)$, as a function of time. Confirm your results by graphing your equation in your graphing calculator in the window in which you sketched it.

$d(t) = 400\cos\left(\frac{\pi}{5}(t+1)\right) - 600$

$d(t) = -400\sin\left(\frac{\pi}{5}(t-1.5)\right) - 600$

$d(t) = -400\cos\left(\frac{\pi}{5}(t-4)\right) - 600$

$d(t) = 400\sin\left(\frac{\pi}{5}(t-6.5)\right) - 600$

√3 √4

(c) Your submarine can't communicate with ships when it is deeper than 500 meters below the surface. When you first made contact with the enemy destroyer, could your submarine communicate? In at least 2 sentences, explain how you arrived at your answer.

$d(t) = -500$

$d(0) = -276.393$ m √5

When contact was first made, we COULD communicate. When $t = 0$ min (at first contact), our sub was only 276.393 meters below the surface, which is within the first 500 meters where communication is possible √6

(d) In the first 12 minutes, for how many total minutes may your submarine communicate with ships. Show the work that leads to your answer and AVOID ANY ROUNDING ERROR!!

$d(t) = -500$ or $d(t) \geq -500$ (√7)

$t = 1.097$ min $= A$
(1st 1.097 min)

$t = 6.902$ min $= B$
$t = 11.097$ min $= C$
(between these 2 times)

So, in the first 12 minutes, we can communicate for $A + (C - B) =$ 5.293 min or 5.294 min (√8)

(e) At what time, in minutes, is your submarine 700 meters below surface going up for the second time?

$d(t) = -700$

$t = 16.097$ min or 16.098 min (√9)

* No penalty points for units this time