

TEST: 5.1-5.5—Calculator Permitted

Part I: Multiple Choice

B 1. Approximately how many cycles will the function $y = 15 + 2 \cos\left(\frac{5\pi}{3} + \frac{2\pi}{7}x\right)$ have between $[0, 2\pi)$?

(A) $\frac{5\pi}{3}$

(B) $\frac{2\pi}{7}$

(C) 7

(D) $\frac{6}{5}$

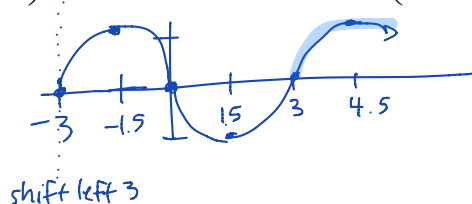
(E) 15

$B = \frac{2\pi}{7}$

C 2. Which of the following is equal to $f(x) = A \sin\left(\frac{\pi}{3}(x+3)\right) + D$?

(A) $f(x) = A \sin\left(-\frac{\pi}{3}(x+3)\right) + D$ (B) $f(x) = A \cos\left(-\frac{\pi}{3}(x+3)\right) + D$ (C) $f(x) = -A \sin\left(-\frac{\pi}{3}(x-3)\right) + D$

(D) $f(x) = -A \cos\left(-\frac{\pi}{3}x\right) + D$ (E) $f(x) = A \cos\left(-\frac{\pi}{3}x\right) + D$



$$P = \frac{2\pi}{1} \left(\frac{3}{\pi}\right) = 6$$

$$\frac{1}{4}P = \frac{6}{4} = \frac{3}{2} = 1.5$$

$$= -(-)A \sin\left(\frac{\pi}{3}(x-3)\right) + D$$

(ODD FUNCTION)

$$= A \sin\left(\frac{\pi}{3}(x-3)\right) + D$$

A 3. Determine the phase shift of the function $f(x) = -2 \sin\left(2\pi - \frac{2x}{3}\right) - \frac{\pi}{3}$

(A) 3π to the right

(B) $\frac{\pi}{3}$ to the right

(C) 2π to the left

(D) 3π to the left

(E) $\frac{\pi}{3}$ to the left

$$= -2 \sin\left(-\frac{2}{3}x + 2\pi\right) - \frac{\pi}{3}$$

$$= -2 \sin\left(-\frac{2}{3}\left(x - 3\pi\right)\right) - \frac{\pi}{3}$$

$\uparrow$
 3π to the right

D 4. The number of solutions to the equation $\sin(30x) = -2$ in the interval $[0, 2\pi]$ is

(A) 30

(B) 60

(C) 2

$-1 \leq \sin(Bx) \leq 1$

(D) 0

(E) $\frac{\pi}{15}$

A 5. Which of the following angles is coterminal with $\frac{-3327\pi}{11}$?

- (A) $\frac{17\pi}{11}$ (B) $\frac{6\pi}{11}$ (C) $\frac{5\pi}{11}$ (D) $\frac{6\pi}{22}$ (E) $\frac{5\pi}{22}$

$$\begin{aligned} -3327/22 &= -151.227... \text{ rotations} \\ &\quad +151 \\ &\quad \underline{-0.227} \\ &\quad \quad \times 22 \\ &\quad \quad \underline{-5} \leftarrow \text{new coefficient} \\ &\quad \quad \quad \text{of } \frac{\pi}{11} \end{aligned}$$

$$\text{So } \frac{-3327\pi}{11} \ominus -\frac{5\pi}{11} \ominus -\frac{5\pi}{11} + \frac{22\pi}{11} = \boxed{\frac{17\pi}{11}} \quad (1 \text{ rotation} = 2\pi)$$

D 6. What is the period of the following sinusoid: $y = 5 - 3 \sin\left(\frac{5\pi}{9} + \frac{3\pi}{7}x\right)$?

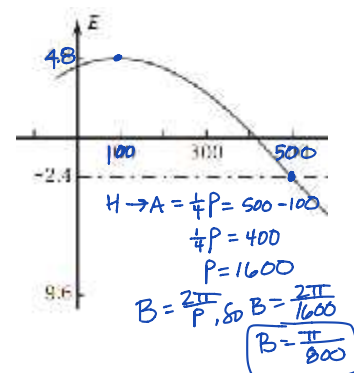
- (A) $\frac{3\pi}{7}$ (B) $\frac{5\pi}{9}$ (C) $\frac{9}{5\pi}$ (D) $\frac{14}{3}$ (E) $\frac{18}{5}$

$$B = \frac{3\pi}{7}$$

$$P = \frac{2\pi}{3\pi/7} = \frac{2\pi}{1} \cdot \frac{7}{3\pi} = \frac{14}{3}$$

B 7. The function $f(x) = A \cos(B(x-C)) + D$ is shown at right. What is the value of B in the equation for $f(x)$?

- (A) $\frac{\pi}{400}$ (B) $\frac{\pi}{800}$ (C) $\frac{\pi}{1600}$ (D) -2.4 (E) 400



B 8. A battery-powered Barbie Ferris Wheel in a Precal student's playroom, has a radius of 55 inches and takes 25 seconds for one complete revolution. This Precal student puts his Ken doll on the Barbie Wheel at a low point when $t = 0$ and is 15 inches above the ground. What is the linear velocity, in miles per hour, of the Ken doll as he travels round and round on the Barbie Wheel?

- (A) 0.392 mph (B) 0.785 mph (C) .9424 mph (D) 0.25 mph (E) 1.041 mph

$$\frac{1 \text{ rev}}{25 \text{ sec}} \times \frac{\overset{\text{Circumference}}{2\pi(55 \text{ in})}}{1 \text{ rev}} \times \frac{60 \text{ sec}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ mile}}{5280 \text{ ft}} = 0.785 \text{ mph}$$

Part II: Free Response

Show all work below the line. Give simplified, exact answers when specified, otherwise report **three decimals**. Avoid intermediate rounding error. Box your final answers, **with units** when appropriate.



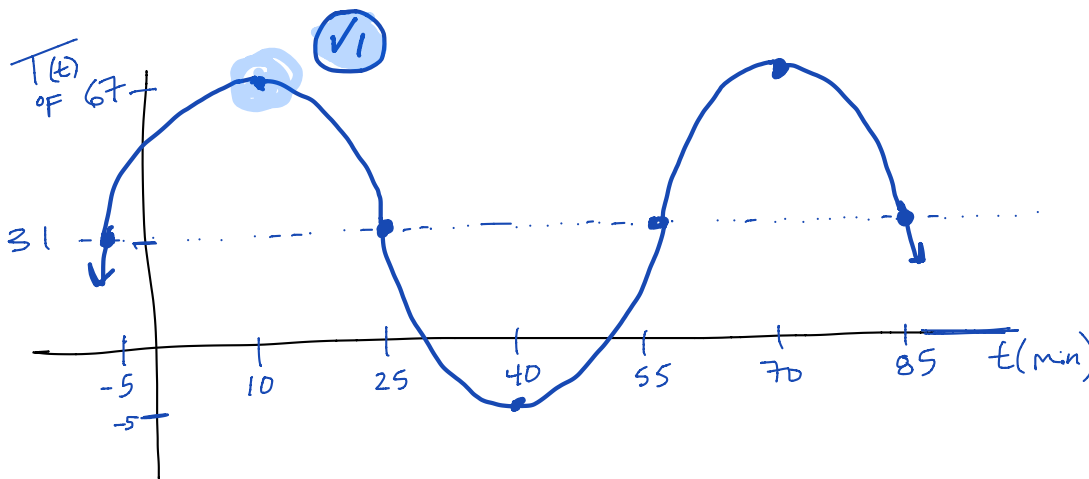
9. **Extraterrestrial Being Problem:** Researchers find a creature from an alien planet. Its body temperature varies sinusoidally with time. Ten minutes after they start timing, the alien's temperature reaches a high of 67°F . 30 minutes after that, it reaches its first low of -5°F .

- (a) Sketch a graph of the sinusoid, graphing the alien's temperature $A(t)$ in degrees Fahrenheit, $^\circ\text{F}$, against time, t , in minutes. Show only the relevant information, including high, axis, low, and critical points. Label all information, including axes. Sketch at least one cycle and at least one critical value to the left of the y -axis.

$$\frac{1}{4}P = 15 \text{ sec} \quad \begin{array}{ccccccccc} -5 & 10 & 25 & 40 & 55 & 70 & 85 \\ A & H & A & L & A & H & A \end{array}$$

$$67^\circ\text{F} \quad -5^\circ\text{F}$$

$$\frac{67 + (-5)}{2} = \frac{62}{2} = 31$$



- (b) Write an equation in standard transformation form for the temperature $A(t)$ of the alien in degrees Fahrenheit against time, t in minutes.

$$P = 60 \quad B = \frac{\pi}{30}$$

$$\text{or } A(t) = 36 \sin\left(\frac{\pi}{30}(t+5)\right) + 31$$

$$\text{or } A(t) = 36 \cos\left(\frac{\pi}{30}(t-10)\right) + 31$$

$$\text{or } A(t) = -36 \sin\left(\frac{\pi}{30}(t-25)\right) + 31$$

$$\text{or } A(t) = -36 \cos\left(\frac{\pi}{30}(t-40)\right) + 31$$

(c) What was the alien's temperature when they first started timing at $t = 0$?

$$A(0) = 49^{\circ}\text{F} \quad (\& \text{ rising})$$

✓4

(d) After how many minutes is the Alien's temperature 0°F and falling for the second time. Show the work that leads to your answer.

$$A(t) = 0 \quad \checkmark 5$$

$$\checkmark 6 \quad t = 94.906 \text{ minutes}$$

or

$$t = 94.907$$

(e) During the first 5 hours, for how many minutes was the alien's temperature above freezing (32°F)? Show the work that leads to your answer and AVOID ANY ROUNDING ERROR!!

$$A(t) > 32 \text{ or } A(t) = 32 \quad \text{Shrs} = 300 \text{ min}$$

$$\left[\begin{array}{l} t = 24.734 \dots \text{min} = A \\ t = 55.2652 \dots \text{min} = B \\ t = 84.734 \dots \text{min} = C \\ t = 295.265 \dots \text{min} = D \end{array} \right. \left. \begin{array}{l} \text{So, total time above freezing} \\ \text{is } A + 4(C - B) + (300 - D) \\ 147.347 \text{ min} \end{array} \right. \quad \checkmark 7$$

(f) If the researchers would have started their timer when the alien's temperature was 10°F and falling for the first time, **by how many minutes** would you have to horizontally shift your graph from part (a) to match the new stopwatch? Would you shift your graph from part (a) to the **left or the right**? Show the work that leads to your answer.

$$A(t) = 10$$

$$\checkmark 8 \quad t = 30.947 \text{ min}$$

or

$$t = 30.948$$

you would have to
shift your graph
LEFT 30.9475

UNITS ✓9