Name KEY Date Period

TEST: 5.1-5.5 C—Calculator Permitted

Part I: Multiple Choice

E 1. What is the period of the following sinusoid: $y = 5 - 3\sin\left(\frac{3\pi}{7} + \frac{5\pi}{9}x\right)$?

(A) $\frac{3\pi}{7}$ (B) $\frac{5\pi}{9}$ (C) $\frac{9}{5\pi}$ (D) $\frac{14}{3}$ (E) $\frac{18}{5}$

$P = \frac{2\pi}{1}\left(\frac{9}{5\pi}\right) = \frac{18}{5}$

A 2. Which of the following angles is coterminal with $\frac{-97531\pi}{11}$?

(A) $\frac{17\pi}{11}$ (B) $\frac{6\pi}{11}$ (C) $\frac{5\pi}{11}$ (D) $\frac{6\pi}{22}$ (E) $\frac{5\pi}{22}$

$-\frac{5\pi}{11} + \frac{22\pi}{11} = \frac{17\pi}{11}$

A 3. Which of the following is equal to $f(x) = -4\sin\left(\frac{\pi}{3}(x-1)\right) - 4$? $P = \frac{2\pi}{1}\left(\frac{3}{\pi}\right) = 6$, $\frac{1}{4}P = \frac{6}{4} = 1.5$

(A) $f(x) = 4\cos\left(\frac{\pi}{3}\left(x + \frac{1}{2}\right)\right) - 4$ (B) $f(x) = 4\cos\left(\frac{\pi}{3}\left(x - \frac{1}{2}\right)\right) - 4$ (C) $f(x) = 4\cos\left(\frac{\pi}{3}\left(x - \frac{1}{4}\right)\right) - 4$

(D) $f(x) = 4\sin\left(\frac{\pi}{3}\left(x - \frac{1}{2}\right)\right) - 4$ (E) $f(x) = -4\sin\left(\frac{\pi}{3}\left(x + \frac{1}{2}\right)\right) - 4$

-.5 1 2.5 4 5.5

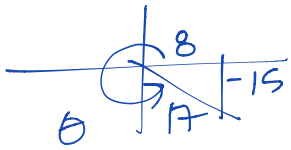
H A L A H

C 4. The number of solutions to the equation $\sin(300x) = -\frac{2}{3}$ in the interval $[0, 2\pi]$ is

(A) 150 (B) 300 (C) 600 (D) 800 (E) 900

B 5. If the terminal ray of $\theta$ passes through the point $(8,-15)$, then $\sin\theta$=?

(A) $\frac{8}{17}$ (B) $-\frac{15}{17}$ (C) $-\frac{8}{15}$ (D) $-\frac{15}{8}$ (E) $\frac{17}{8}$




$\sin\theta = -\frac{15}{17}$

D 6. Approximately how many cycles will the function $f(x)=5\cos\frac{3\pi}{4}x$ have from 0 to $2\pi$?

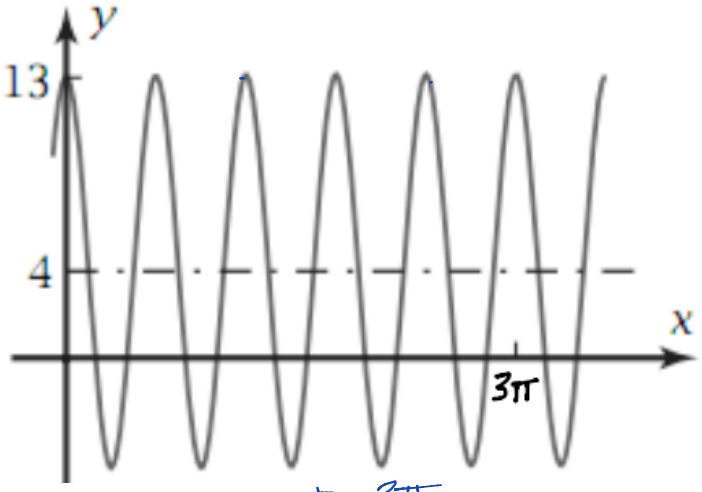
(A) 2.667 (B) 0.75 (C) 5 (D) 2.356 (E) 0.424

$B = \frac{3\pi}{4} = 2.356$

E 7. The function $f(x)=A\cos(B(x-C))+D$ is shown at right. What is the value of $B$ in the equation for $f(x)$?

(A) $\frac{3\pi}{5}$ (B) $3\pi$ (C) $\frac{3}{2}$

(D) $\frac{2}{3}$ (E) $\frac{10}{3}$




$P = \frac{3\pi}{5}$

$B = \frac{2\pi}{1}\cdot\frac{5}{3\pi} = \frac{10}{3}$

D 8. Determine the range of the function $y=\frac{b}{2}\cos 4ax-\frac{3b}{2}$, where $a>0$, $b>0$.

(A) $\left\{y\middle|\frac{b}{2}\le y\le\frac{3b}{2}\right\}$ (B) $\left\{y\middle|-\frac{3b}{2}\le y\le-\frac{b}{2}\right\}$ (C) $\{y|b\le y\le 2b\}$

(D) $\{y|-2b\le y\le -b\}$ (E) $\{y|-b\le y\le 2b\}$

$\left[-\frac{3b}{2}-\frac{b}{2}, -\frac{3b}{2}+\frac{b}{2}\right]$

$\left[-\frac{4b}{2}, -\frac{2b}{2}\right]$

$[-2b, -b]$

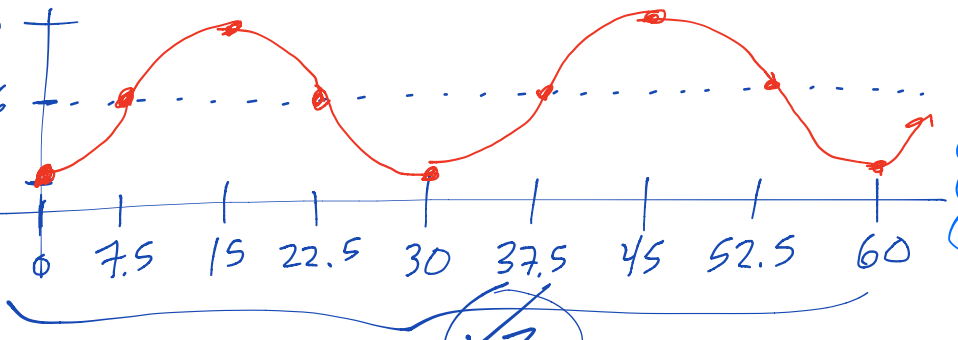
Part II: Free Response

Show all work below the line. Give simplified, exact answers when specified, otherwise report **three decimals**. SHOW ALL CALCULATOR SETUPS. Avoid intermediate rounding error. Box your final answers, **with units** when appropriate.

9. The London Eye is a giant Ferris wheel on the South Bank of the River Thames in London. The diameter of the wheel is 394 feet. As the London Eye makes one rotation every 30 minutes, its riders reach a maximum height of 443 feet above ground. As a person gets on the constantly rotating ride at the bottom, he starts his stopwatch.

(a) Sketch the graph of the rider's height above ground $h$ in feet as a function of time $t$ in minutes since the rider started the stopwatch. **Sketch two full cycles of the ride, labeling your critical values and axes.**

$|A| = 197$
$P = 30$
$B = \frac{\pi}{15}$
$D = 246$
$R: [49, 443]$
✓1




✓2

1 E 5 B
2 A 6 D
3 A 7 E
4 C 8 D

(b) Assuming that $h$ is a sinusoidal function of time $t$, write a particular equation for $h(t)$. Confirm your results by graphing your equation in your graphing calculator in the window in which you sketched it.

$h(t) = -197\cos\left(\frac{\pi}{15}t\right) + 246$ ✓3

(c) How high above the ground is the rider 24.3 minutes into the ride? At this time, is the rider going up or down?

$h(24.3) = 173.479$ ft going down

✓4

(d) At what positive time, in minutes, is the rider 400 feet above ground coming down for the second time?

$h(t) = 400$

$t = 48.215 \text{ sec}$ ✓5

(e) Assuming the rider stays on for 2 full rotations, for how many minutes was the rider above 430 feet? Show the work that leads to your answer.

$h(t) = 430$ or $h(t) > 430$

✓6 $\begin{cases} t_1 = 13.255 \text{ min} = A \\ t_2 = 16.744 \text{ min} = B \end{cases}$ ✓7

Total time $= 2(B - A) = 6.977 \text{ min}$

(f) Through how many feet does the rider travel during 20 minutes?

$S = r\theta$

$= (197 \text{ ft})\left(\frac{20}{30}(2\pi)\right)$

$= \frac{788\pi}{3} \text{ ft}$ or $262.666\pi$ or $825.191 \text{ ft}$ or $825.192 \text{ ft}$ ✓8

(g) What is the linear velocity of the rider, **in miles per hour**, as the Ferris Wheel rotates?

$V = r\omega$

$= (197 \text{ ft})\left(\frac{1 \text{ rot}}{30 \text{ min}}\right)\left(\frac{1 \text{ mil}}{5280 \text{ ft}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rot}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)$

$= 0.468 \text{ mph}$ or $0.149\pi \text{ mph}$

✓9 $0.469 \text{ mph}$ or $\frac{197\pi}{1320} \text{ mph}$

9 checks

(h) Xtra Credit: If the rider drops an apple from the very top of the ride, how long (in seconds) will it take for the apple to hit the ground below? Show the calculations/formulas that lead to your answer.

$h(t) = -\frac{1}{2}gt^2 + V_0 t + h_0$

$h(t) = -16t^2 + 443$

Let $h(t) = 0$

$0 = -16t^2 + 443$

$443 = 16t^2$

$t^2 = \frac{443}{16}$

$t = \sqrt{\frac{443}{16}} \text{ sec}$

$t = 5.261 \text{ sec}$

$t = 5.262 \text{ sec}$

17 checks total