Name_____ Date_____ Period_____

TEST: 5.1-5.5—Calculator Permitted

Part I: Multiple Choice

C 1. What is the period of the following sinusoid: $y = 5 - 3\cos\left(\frac{2\pi}{3} + \frac{4\pi}{3}x\right)$?

(A) $\frac{2\pi}{3}$ (B) $\frac{4\pi}{3}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

E 2. Which of the following angles is coterminal with $\frac{447755\pi}{7}$?

(A) $\frac{3\pi}{7}$ (B) $\frac{5\pi}{7}$ (C) $\frac{9\pi}{7}$ (D) $\frac{11\pi}{7}$ (E) π

E 3. Which of the following is equal to $\sec^{-1} 0.5$?

(A) 0.524 (B) 0.016 (C) 0.954 (D) 1.139 (E) undefined

B 4. For $\theta = -16115123.2^\circ$, Find the reference angle, θ_{ref}

(A) 6.798° (B) 83.202° (C) -83.202° (D) -6.798° (E) $\frac{\pi}{6}$

C 5. If the terminal ray of θ passes through the point $(-1,-2)$, then $\csc\theta = ?$

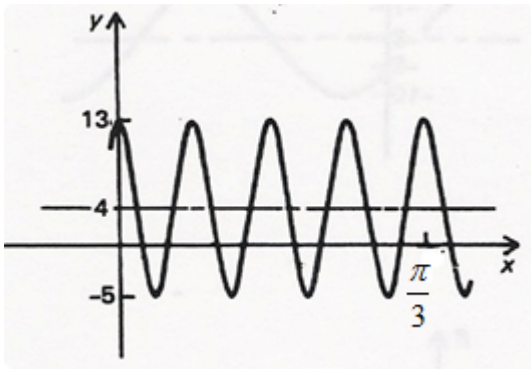
(A) $\frac{1}{2}$ (B) 2 (C) $-\frac{\sqrt{5}}{2}$ (D) $-\sqrt{5}$ (E) $-\frac{\sqrt{5}}{5}$

$\frac{-1}{-2}$ $\frac{1}{\sqrt{5}}$

A 6. Approximately how many cycles will the function $f(x) = 13\sin(4\pi x) - 14$ have from 0 to 2π?

(A) 12.5 (B) 4 (C) 2 (D) 0.5 (E) 0.25

C 7. For the given graph shown at right, how many cycles does it have between 0 and 2π?



(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) 24 (D) 12 (E) 36

D 8. Determine the range of the function $y = -\frac{3b}{2} + \frac{b}{2}\cos 4ax$, where $a > 0$, $b > 0$.

(A) $\left\{y \middle| \frac{b}{2} \le y \le \frac{3b}{2}\right\}$ (B) $\left\{y \middle| -\frac{3b}{2} \le y \le -\frac{b}{2}\right\}$ (C) $\{y | b \le y \le 2b\}$

(D) $\{y | -2b \le y \le -b\}$ (E) $\{y | -b \le y \le 2b\}$

$\left[-\frac{3b}{2} - \frac{b}{2}, -\frac{3b}{2} + \frac{b}{2}\right]$

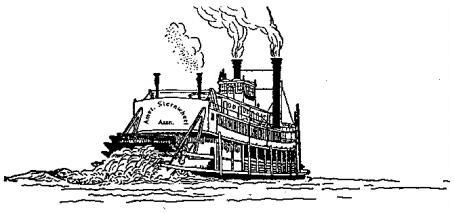
$\left[-\frac{4b}{2}, -\frac{2b}{2}\right]$

$[-2b, -b]$

Part II: Free Response

Show all work, including the equations your are solving/evaluating on your calculator. Give simplified, exact answers when specified, otherwise report **three decimals**. Avoid intermediate rounding error. Box your final answers, **with units** when appropriate.

Mark Twain sat on the deck of a river steamboat with his stopwatch. As the paddlewheel turned, **a point** on the outer edge of the paddlewheel moved in such a way that its distance, d in **feet**, from the water's surface was a sinusoidal function of time, t in **seconds**. When his stopwatch read 4 seconds, the point was at its highest, 16 ft above the water's surface. The wheel's diameter was 18 ft and rotates at 6 RPMs (revolutions per minute).



(a) What is the period of the paddle wheel?

(b) Sketch at least 2 cycles of the graph of the sinusoid. Be sure to label and scale both your axes. Show it crossing the y-axis.

(c) Write an equation of your graph modeling the height, $d(t)$, at time t.

(d) Where was the point, in feet, **in relation to the water's surface** when Mark started his stopwatch? At this time, in which vertical direction was that point moving?

(e) During the first 15 seconds from the time Mark started his watch, for how many seconds does the point remain underwater? Show the work that leads to your answer.

(f) What is the total distance, in feet, this point on the outer edge of the paddlewheel travel in one minute?

(g) What is the linear velocity, in feet per second, of this point?

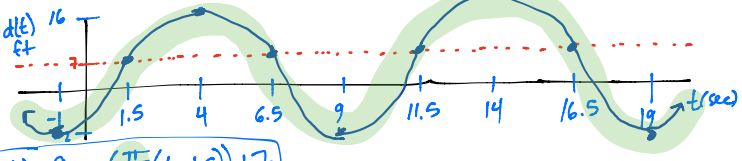
(h) What was the full birth name of Mark's grandfather's son's son's name?

CEEBCACD

(a) $\frac{6\text{ Rev}}{1\text{ min}} = \frac{1\text{ rev}}{\frac{1}{6}\text{ min}}$

$P = \frac{1}{6}$ min or 10 sec

(b) $d(t)$ ft



(c) $d(t) = 9\sin\left(\frac{\pi}{5}(t-1.5)\right)+7$

(d) $d(0) = -0.281$ ft

∴ 0.281 ft below water and rising

(e) $d(t) = 0$

$t_1 = 0.0817...$ sec $= A$

$t_2 = 7.918...$ sec $= B$

$t_3 = 10.0817...$ sec $= C$

Total Time $= A + (C-B)$

$= 2.245$ sec ← 2 checks

Note $(C-B)$ ~~Lost of 2~~ if only $= 2.163$ sec

(f) $S = r\theta$

$S = (9\text{ ft})(12\pi\text{ rad}) = 108\pi$ ft or 339.292 ft

In 1 min, wheel turns 6 times

$\theta = (2\pi)6 = 12\pi$ rad

(g) $V = r\omega$

$V = (9\text{ ft})\left(\frac{6\text{ rev}}{1\text{ min}}\right)\left(\frac{1\text{ min}}{60\text{ sec}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)$

$V = 1.8\pi$ ft/sec or $\frac{9\pi}{5}$ ft/sec or 5.6548 ft/sec

(h) Samuel Langhorne Clemens