Name Key Date 3/2/20 Period 18 points

TEST: 5.1 – 5.3 II with Angular Velocity and Unit Circle—Calculator Permitted

Part I: Multiple Choice—Put the CAPITAL letter of the correct answer in the blank.

E 1. $\csc(123456789\pi) =$

calculator might give this answer due to decimal overflow! $-251{,}962.295 \approx -\infty$ LOL!

(A) -3.966 (B) -251962.295 (C) 0.977 (D) -1 (E) DNE

Radian mode

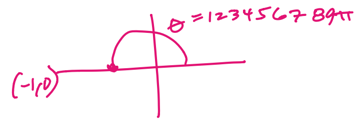
$$\frac{1}{\sin(123456789\pi)}$$

$$\frac{1}{0}$$

DNE

Note

$123456789\pi \;\cancel{\theta}\; \pi$ (odd multiple of π)



A 2. What is the reference angle for $\theta = \dfrac{6703\pi}{51}$?

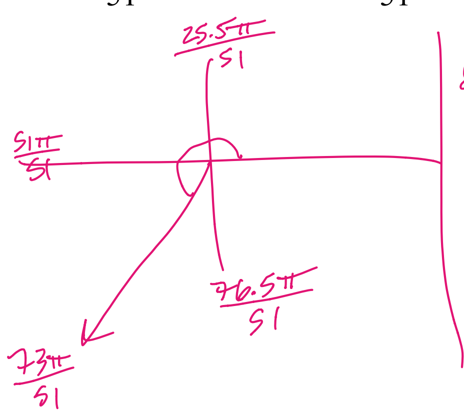
(A) $\dfrac{22\pi}{51}$ (B) $\dfrac{19\pi}{51}$ (C) $\dfrac{73\pi}{51}$ (D) $\dfrac{3\pi}{51}$ (E) $\dfrac{7\pi}{102}$

$6703/102 = 65.7156\ldots$ rotations

$65.715\ldots - 65 = 0.7156\ldots$

$0.7156\ldots \times 102 = 73$

So, $\dfrac{6703\pi}{51} \;\cancel{\theta}\; \dfrac{73\pi}{51}$



So, $\theta_{ref} = \dfrac{73\pi}{51} - \dfrac{51\pi}{51}$

$= \dfrac{22\pi}{51}$

A 3. For $\theta = -998877^\circ$, find the coterminal angle β for θ, such that $0^\circ \le \beta < 360^\circ$

(A) 123° (B) 237° (C) 2774° (D) 57° (E) 303°

$-998877 \div 360 = -2774.658\ldots$ rotations

$-2774.658 + 2774 = -0.658\ldots$ rotation

$-0.658\ldots \times 360 = -237^\circ$

$-237^\circ + 360^\circ = 123^\circ$

D ____ 4. $\cot\left(\dfrac{-2\pi}{3}\right)=$

(A) $\sqrt{3}$ (B) $-\sqrt{3}$ (C) $-\dfrac{\sqrt{3}}{3}$ (D) $\dfrac{\sqrt{3}}{3}$ (E) DNE

$\cot\left(-\frac{2\pi}{3}\right)$ ← add 1 rotation

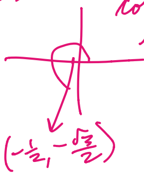
$\cot\left(-\frac{2\pi}{3}+\frac{6\pi}{3}\right)$

$\cot\left(\frac{4\pi}{3}\right)$

$\frac{-1}{-\sqrt{3}}$

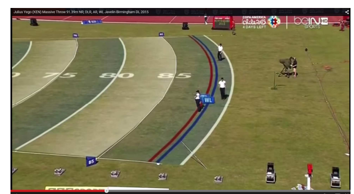
$\frac{1}{\sqrt{3}}$

$\frac{\sqrt{3}}{3}$



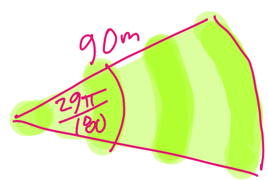
*coterminal angles have the same trig ratios!

E ____ 5. The javelin throw is a common field event. If the landing area is the shape of a sector of a circle with a $29°$ central angle, approximately how much area, in square meters, does a javelin have to land in if the radius of the landing field is 90 meters?



(A) 4100 (B) 117450 (C) 234900 (D) 519 (E) 2050

$\frac{29°}{1} \times \frac{\pi}{180°} = \frac{29\pi}{180}$ rads $= \theta$



$A = \frac{1}{2}r^2\theta$

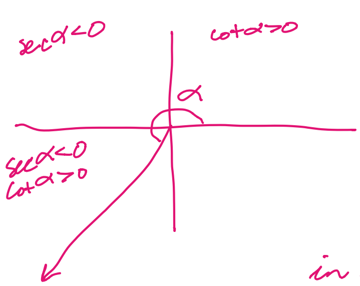
$= \frac{1}{2}(90^2)\left(\frac{29\pi}{180}\right)$

$= 2049.889\ldots\ m^2$

$\approx 2050\ m^2$

D ____ 6. If $\sec\alpha = -\dfrac{11}{5}$ and $\cot\alpha > 0$, which of the following could be α

(A) $117.035°$ (B) $62.964°$ (C) $-422.964°$ (D) $602.964°$ (E) $1197.035°$



if $\sec\alpha = -\frac{11}{5}$

then $\cos\alpha = -\frac{5}{11}$

$\alpha = \cos^{-1}\left(-\frac{5}{11}\right)$

in degrees: $\alpha = 117.035\ldots$ ← QII

so, $\alpha_{ref} = 180 - 117.035\ldots$

$\alpha_{ref} = 62.964°\ldots$

& $\alpha = 180° + \alpha_{ref}$

$= 180° + 62.964\ldots°$

$= 242.964\ldots°$

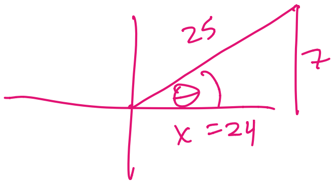
$242.964\ldots°$ is not an answer choice, so the angle α must be coterminal with 242.964. Here is a list formed by adding & subtracting 360°.

$\ldots, -477.035°, -117.035°, 242.964°, 602.964°, 962.964°, \ldots$

Whoop! There it is!

C ______ 7. If $\sin\theta = \frac{7}{25}$ and $\theta \in \left(0, \frac{\pi}{2}\right)$, what is the value of $\sec\theta$?

(A) $\frac{24}{7}$ (B) $\frac{25}{7}$ (C) $\frac{25}{24}$ (D) $\frac{24}{25}$ (E) $\frac{7}{24}$



$\sin\theta = \frac{7}{25} = \frac{y}{r}$

$x^2 + 7^2 = 25^2$

$x = \pm 24$

So, $x = 24$

$\sec\theta = \frac{r}{x}$

$= \frac{25}{24}$

B ______ 8. A school bus full of Unicorns is cruising down IH-35 on its way to a math competition. If the bus has wheels 42 inches in diameter that spin at 384 RPMs, approximately how fast is the bus cruising down the interstate highway?

(A) 43 mph (B) 48 mph (C) 53 mph (D) 57 mph (E) 62 mph

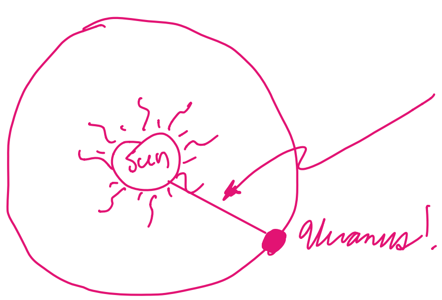
$\frac{384\ \cancel{rev}}{1\ \cancel{min}} \times \frac{60\ \cancel{min}}{1\ hr} \times \frac{42\pi\ \cancel{in}}{1\ \cancel{rev}} \times \frac{1\ \cancel{ft}}{12\ \cancel{in}} \times \frac{1\ mil}{5280\ \cancel{ft}}$

47.9806 mph

≈ 48 mph

B ______ 9. Uranus travels in a nearly-circular orbit around the sun, which is approximately 1,800,000,000 miles away. If it makes it around the sun once every 84 Earth years, approximately how fast is Uranus moving in its orbit through outer space?

(A) 9,000 mph (B) 15,000 mph (C) 23,000 mph (D) 42,000 mph (E) 57,000 mph



r = 1,800,000,000 miles

$V = r \cdot \omega$

$= (1800000000\ mil)\left(\frac{1\ \cancel{rot}}{84\ \cancel{yrs}}\right)\left(\frac{2\pi\ rad}{1\ \cancel{rot}}\right)\left(\frac{1\ \cancel{yr}}{365\ \cancel{days}}\right)\left(\frac{1\ \cancel{day}}{24\ hr}\right)$

$= 15369{,}827\ldots$ mph

≈ 15370 mph

$\approx 15{,}000$ mph

Part II: Free Response

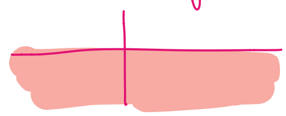
Show all work below. Avoid intermediate rounding error. Box your final answers, with units when appropriate. NOTE: the following questions are unrelated to each other and may be answered independently of each other.

10. Use the Unit Circle and your trigonometric knowledge to answer the following questions.

(a) In which two quadrants is cosecant negative?

$\csc\theta = \frac{r}{y}$,

So, $\csc\theta < 0$ when $y < 0$

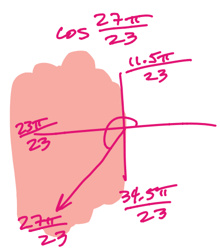


$\boxed{\text{QIII \& QIV}}$ √1 √2

(b) If $\cos\frac{27\pi}{23} = \cos\beta$, what is the simplified, exact value of $\beta \in (0, 2\pi)$, where $\beta \neq \frac{27\pi}{23}$?

$\cos\theta < 0$

So, $\cos\theta < 0$ when $x > 0$



the other quadrant where $\cos\theta < 0$ is QII.

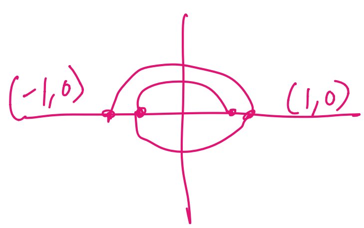
$\beta_{ref} = \frac{27\pi}{23} - \frac{23\pi}{23}$

$\boxed{\beta_{ref} = \frac{4\pi}{23}}$ √3

So, $\beta = \frac{23\pi}{23} - \frac{4\pi}{23}$

$\boxed{\beta = \frac{19\pi}{23}}$ √4

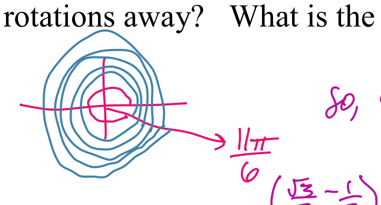
(c) Which two trig functions are undefined when a terminal ray lands on the *x*-axis?



Such angles have reference triangles with no height ($y = 0$), So, the 2 trig functions with y in the denominator

or $\boxed{\frac{x}{y} = \cot\theta \;\&\; \frac{r}{y} = \csc\theta}$ √5 √6

(d) If $\theta = \frac{11\pi}{6}$, what is the simplified, exact value of a coterminal angle, α, to θ that is 5 positive rotations away? What is the simplified, exact value of $\cot\alpha$?



So, $\alpha = \frac{11\pi}{6} + 5(2\pi)$

$= \frac{11\pi}{6} + 10\pi$

$= \frac{11\pi}{6} + \frac{60\pi}{6}$

$\boxed{\alpha = \frac{71\pi}{6}}$ √7

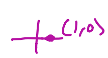
$\boxed{\cot\alpha = \cot\frac{11\pi}{6} = \frac{\sqrt{3}}{-1} = -\sqrt{3}}$ √8

(e) If $\cos\frac{3\pi}{2} = A$, what is $\tan(-A)$?

$\cos\frac{3\pi}{2} = 0$

So, $A = 0$

$\tan(-A) = \tan(-0)$



$= \tan 0$

$= \frac{0}{1}$

$= \boxed{0}$ √9