Name KEY Date 2/22/17 Period !

TEST: 5.1 – 5.3—Calculator Permitted
Angles, angle measure, applications of angles, & Circular Functions.

Part I: Short Answer—Show all work. No work, no credit.

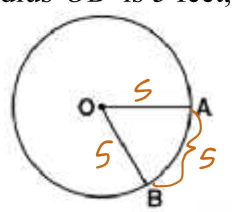
D 1. The terminal side of an angle in standard position lies in quadrant IV of the coordinate grid. The radian measure of this angle could be which of the following?

(A) $\frac{6\pi}{5}$ (B) $\frac{3\pi}{4}$ (C) $-\frac{2\pi}{3}$ (D) $-\frac{7\pi}{3}$ (E) $\frac{16\pi}{11}$

$-\frac{7\pi}{3} = -\frac{6\pi}{3} - \frac{\pi}{3}$

$-7\pi/3$

A 2. In circle $O$ below, the length of the radius $\overline{OB}$ is 5 feet, and the length of arc $\overset{\frown}{AB}$ is 5 feet.




$\theta = \frac{s}{r} = \frac{5ft}{5ft} = 1 rad$

The measure of central angle $\angle AOB$ is which of the following?

(A) 1 radian (B) 60° (C) greater than 60° (D) $\pi$ radians (E) 5 radians

D 3. Through how many radians does the minute hand of a clock turn in 48 minutes?

(A) $\frac{6\pi}{5}$ (B) $\frac{7\pi}{5}$ (C) $\frac{9\pi}{5}$ (D) $\frac{8\pi}{5}$ (E) $\frac{4\pi}{5}$

48 min is $\left(\frac{48}{60} \text{ rotations}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rot}}\right)$

$= \frac{48\pi}{30} = \frac{24\pi}{15} = \frac{8\pi}{5}$ rads

E 4. $\sec\left(\frac{-47537\pi}{6}\right) = \sec\left(\frac{-5\pi}{6}\right) = \sec\left(\frac{7\pi}{6}\right) = \frac{-2\sqrt{3}}{3}$

(A) $-\frac{\sqrt{3}}{2}$ (B) $\frac{2\sqrt{3}}{3}$ (C) $-\frac{1}{2}$ (D) $-2$ (E) $-\frac{2\sqrt{3}}{3}$

$-47537/12 = -3961.416$

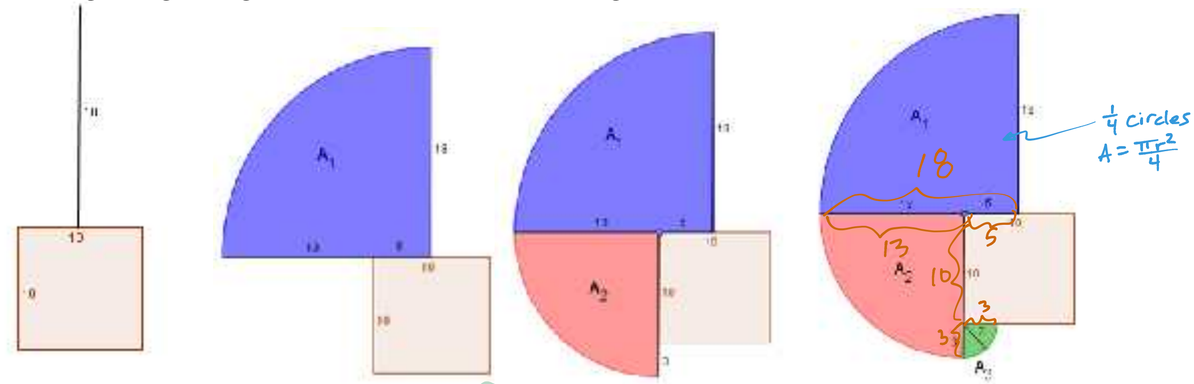
C 5. The platform of a large merry-go-round is 200 feet in **diameter**. To the nearest **mile per hour**, how fast does a person standing on the outer edge of the platform travel if the merry-go-round makes 6 revolutions per minute? (Hint: someone who's actual foot is actually 1 foot long has a foot that is actually $\frac{1}{5280}$ of a mile long!) $V = r\omega$

(A) 50 mph (B) 21 mph (C) 43 mph (D) 214 mph (E) 62 mph

$V = (100 \cancel{ft})\left(\frac{6 \text{ rev}}{1 \text{ min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)\left(\frac{1 \text{ mile}}{5280 \cancel{ft}}\right)$

$= 42.839 \approx 43$ mph

C 6. A goat is tethered to the side of a square 10ft x 10ft shed. The tethering rope is 18 feet long and tied to a post in the middle of one of the sides. To the nearest square foot, what is the **total area** (on both sides) available for the goat to graze? Assume the shed is in the middle of a 100ft x 100ft field. **One side** of the goats grazing area is illustrated in the diagram below.




(A) 394 (B) 502 (C) 789 (D) 1577 (E) 2205

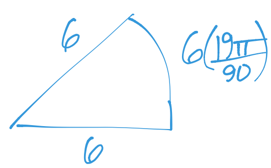
$$A = 2\left[\frac{\pi(18)^2}{4} + \frac{\pi(13)^2}{4} + \frac{\pi(3)^2}{4}\right]$$

$$A = \frac{2\pi}{4}\left[18^2 + 13^2 + 3^2\right] = \frac{\pi}{2}\left[18^2 + 13^2 + 3^2\right] = 788.539 \approx 789\,ft^2$$

A 7. A wedge-shaped piece of pizza is cut from a 12-inch diameter Archimedian Pizza (main topping is sand, perfectly round, & sliced perfectly through the center). The angle measure of the pointy-piece from the center of the pizza measures $38°$. If $A$ is the surface area of the slice and $P$ is the perimeter of the slice, to the nearest whole number, what is the value of $A + P$?

(A) 28 (B) 245 (C) 30 (D) 296 (E) 33

$$\theta = 38°\left(\frac{\pi}{180°}\right) = \frac{19\pi}{90}$$




$P = 6 + 6 + 6\left(\frac{19\pi}{90}\right) =$

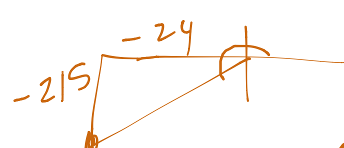
$A = \frac{1}{2}(6^2)\left(\frac{19\pi}{90}\right)$

$A + P = 27.917 \approx 28$

D 8. The terminal ray of an angle $\theta$ passes through the point $(-24, -215)$. If $0° \le \theta < 360°$, what is θ ?

Degree Mode

(A) 186.369° (B) 83.630° (C) 96.369° (D) 263.630° (E) 206.369°




$\theta = \tan^{-1}\left(\frac{215}{24}\right)$

$\theta_{ref} = 83.630°$

$\theta = 180° + 83.630°\ldots$

$\theta = 263.630°$

D 9. If $\sin\theta = -0.4$, then $\sin(-\theta) + \csc\theta =$

(A) 0 (B) $-2.9$ (C) 2.1 (D) $-2.1$ (E) 2.9

$\sin(-\theta) + \csc\theta$

$= 0.4 + \frac{1}{-0.4}$

$= -2.1$

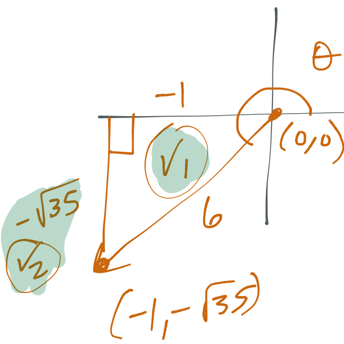
Part II: Free Response

Show all work below. Avoid intermediate rounding error. Box your final answers, with units when appropriate.

10. If $\sec\theta = -6$ and $\cot\theta > 0$

(a) Draw the reference triangle for $\theta$ in the correct quadrant. Show your arc and angle $\theta$.

$\sec\theta = \frac{6}{-1} = \frac{r}{x}$ $\quad \cot\theta > 0$




$y = -\sqrt{36-1}$

$y = -\sqrt{35}$

(b) Find the **simplified, exact, rationalized** value of $\csc\theta$.

$\csc\theta = \frac{r}{y} = \frac{6}{-\sqrt{35}} = \frac{-6\sqrt{35}}{35}$ √3

(c) Find the **simplified, exact, rationalized** value of $\tan\theta$.

$\tan\theta = \frac{y}{x} = \frac{-\sqrt{35}}{-1} = \sqrt{35}$ √4

(d) Find the reference angle, $\theta_{ref}$, for $\theta$ in degrees. **<u>Show the equation you are solving</u>** and report 3 decimals.

$\theta = \tan^{-1}\sqrt{35}$ or $\theta = \cos^{-1}(-1/6)$ $\sqrt{5}$

or $\theta = \sin^{-1}\left(-\frac{\sqrt{35}}{6}\right)$ ... $\theta = 99.594...^\circ$

$\theta_{ref} = 180^\circ - 99.594^\circ...$

$\theta_{ref} = 80.405^\circ$ or $80.406^\circ$ $\sqrt{6}$

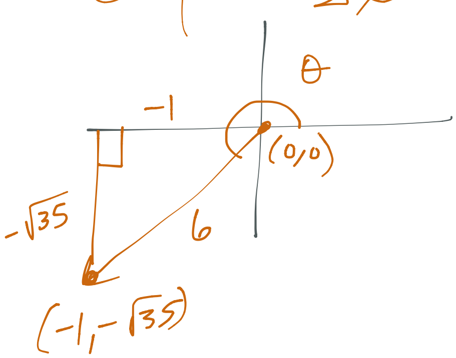
(e) To three decimals, find the value of $\theta$ such that $\theta \in [0^\circ, 360^\circ)$. Show the computations that lead to your answer.

$\theta = 180^\circ + \theta_{ref}$

$\theta = 260.405^\circ$ or $260.406^\circ$ $\sqrt{7}$

(f) In terms of $\theta$, what is the slope of terminal ray?

$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{-\sqrt{35} - 0}{-1 - 0} = \sqrt{35} = \tan\theta$ or $\sqrt{8}$




$\sqrt{9}$ unit check on (d) & (e)