Name KEY Date____________________ Period________

TEST: Chapter 4.3-4.5

I. Multiple Choice: ***(No Calculator)*** Place the capital letter of the answer choice in the blank to the left of the number.

C 1. $(\log_7 5)(\log_3 2)(\log_2 7) =$

(A) $\dfrac{\ln 3}{\ln 2}$ (B) $\log_3 2$ (C) $\log_3 5$ (D) $\log_7 2$ (E) 1

$$\frac{\ln 5}{\cancel{\ln 7}} \cdot \frac{\cancel{\ln 2}}{\ln 3} \cdot \frac{\cancel{\ln 7}}{\cancel{\ln 2}} = \frac{\ln 5}{\ln 3} = \log_3 5$$

A 2. Solve for x: $2^{3x-1} = 5 \cdot 3^{x+1}$

(A) $\dfrac{\ln 30}{\ln 8 - \ln 3}$ (B) $\dfrac{\ln 10}{\ln 6 - \ln 3}$ (C) $\dfrac{\ln 30}{\ln 6 - \ln 2}$ (D) $\dfrac{\ln 10}{\ln 5}$ (E) $\dfrac{\ln 10}{\ln 8 - \ln 3}$

$(3x-1)\ln 2 = \ln 5 + (x+1)\ln 3$

$3x\ln 2 - \ln 2 = \ln 5 + x\ln 3 + \ln 3$

$x(3\ln 2 - \ln 3) = \ln 5 + \ln 3 + \ln 2$

$$x = \frac{\ln 5 + \ln 3 + \ln 2}{3\ln 2 - \ln 3}$$

$$x = \frac{\ln(5 \cdot 3 \cdot 2)}{\ln 2^3 - \ln 3} = \frac{\ln 30}{\ln 8 - \ln 3}$$

D 3. Which of the following is equivalent to the function $\log 72$?

(A) $8\log 9$ (B) $\log 9 \bullet \log 8$ (C) $3\log 3 + 4\log 2$ (D) $2\log 3 + 3\log 2$ (E) $\dfrac{\ln 8}{\ln 9}$

$\log 72$

$\log(9 \cdot 8)$

$\log(3^2 \cdot 2^3)$

$2\log 3 + 3\log 2$

C 4. What is the domain of $y = 5 - 3\ln(-16 + x^2 + 6x)$?

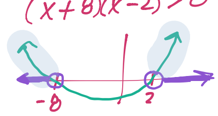
(A) $(-8,2)$ (B) $(-2,8)$ (C) $(-\infty,-8)\cup(2,\infty)$ (D) $(-\infty,-2)\cup(8,\infty)$ (E) $(2,\infty)$

Argument > 0

$-16 + x^2 + 6x > 0$

$x^2 + 6x - 16 > 0$

$(x+8)(x-2) > 0$



$x < -8$ or $x > 2$

D 5. If $\log(x+1)+\log x=\log 6$, then x equals

(A) -3 or 2 (B) 3 or 2 (C) 6 (D) 2 `(E) -3

$\log[(x+1)(x)] = \log 6$

So, $(x+1)(x) = 6$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3$ or $\boxed{x=2}$

extraneous (give a negative argument)

A 6. If $\log_2(2x-4)-\log_2(x-5)=3$, then x equals

(A) 6 (B) 11 (C) 1 (D) All real numbers (E) No solution

$\log_2\left[\frac{2x-4}{x-5}\right]=3$

$2^3 = \frac{2x-4}{x-5}$

$2x-4 = 8(x-5)$

$2x-4 = 8x-40$

$2x-4 = 8x-40$

$2x-8x = 4-40$

$-6x = -36$

$x = 6$

6 works (Gives pos arguments)

B 7. Solve for x by getting the bases the same:

$$\left(\sqrt[3]{3^{15}}\right)\left(\frac{1}{9}\right)^{-x-3} = \left(\frac{\sqrt{3}}{27}\right)^{8x}$$

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 2 (D) -2 (E) $\frac{9}{2}$

$\left(3^{15/3}\right)\left(3^{-2}\right)^{-x-3} = \left(\frac{3^{1/2}}{3^3}\right)^{8x}$

$3^5 \cdot 3^{2x+6} = \left(\frac{3^{4x}}{3^{24x}}\right)$

$3^{5+2x+6} = 3^{4x-24x}$

$3^{2x+11} = 3^{-20x}$

So, $2x+11 = -20x$

$22x = -11$

$x = -\frac{11}{22}$

$x = -\frac{1}{2}$

E 8. If $\log_5 x = A\log_{1/4} x$ use the change of base formula to find the value of A,

(A) $\frac{5}{4}$ (B) $\log_5 4$ (C) 5 (D) -5 (E) $-\log_5 4$

$\left(\frac{\ln 1/4}{\ln x}\right)\frac{\ln x}{\ln 5} = A\cdot\frac{\ln x}{\ln(1/4)}\left(\frac{\ln 1/4}{\ln x}\right)$

$A = \frac{\ln 1/4}{\ln 5}$

$A = \frac{\ln 1 - \ln 4}{\ln 5}$

$A = -\frac{\ln 4}{\ln 5} = -\log_5 4$

II. Free Response: ***(Calculator Permitted)*** **Show all work, including any step, set up, or notation you plug into the calculator.** Work each section in the space provided. **Use correct units** on final answers. Final answers must be accurate to **3 decimal places** (rounded or truncated).

10. **Cooling an Engine**

Suppose you are driving your automobile on a cold winter day ($20°F$ outside) and the engine overheats (to about $220°F$). When you park, the engine begins to cool down. The temperature T of the engine t minutes after you park satisfies the following equation:

↳ in °F

$$t = -9\ln\left(\frac{T-20}{200}\right)$$

(a) Solve the equation for T.

$$t = -9\ln\left(\frac{T-20}{200}\right)$$

✓1 A 1st legit move

$$\frac{t}{-9} = \ln\left(\frac{T-20}{200}\right)$$

$$e^{-t/9} = \frac{T-20}{200}$$

$$200e^{-t/9} = T-20$$

so, $T(t) = 200e^{-t/9} + 20$ ✓2

or just

$$T = 200e^{-t/9} + 20$$

(b) Use your equation from part (a) to find the temperature (to 3 decimals) of the engine after 5 minutes ($t = 5$).

✓3 plug into eq from (a) OR $T(5)$

When $t=5$: $T(5) = 200e^{-5/9} + 20$

function notation

$= 134.750\ °F$

✓4 or

$134.751\ °F$

(c) As your car cools, you call your talkative best friend to come pick you up. After what seems like an eternity, you hang up the phone and find that your car's engine temperature is now 65°F. How many minutes had passed from the time you parked your car to the time you hung up the phone with your chatty chum?

using original, given equation:

$$t(T) = -9\ln\left(\frac{T-20}{200}\right)$$

function notation

or

$$t = -9\ln\left(\frac{T-20}{200}\right)$$

When $T = 65°F$: $t(65) = -9\ln\left(\frac{65-20}{200}\right)$ √5 plug into equation <u>OR</u> $t(65)$

$= 13.424$ min

√6 or

13.425 min

(d) According to the given mathematical model and equation, if you were to leave your car parked in the same place for several days before getting back to it, what would the car's engine temperature be? That is, what is $\lim_{t\to\infty} T$?

$\lim_{t\to\infty} T(t) = 20°F$ √4

units on all parts √8