

16 total checks

I. Multiple Choice: Place the capital letter of the answer choice in the blank to the left of the number.

B 1. $(\log_5 2)(\log_3 7)(\log_7 5) = \frac{\ln 2}{\ln 5} \cdot \frac{\ln 7}{\ln 3} \cdot \frac{\ln 5}{\ln 7} = \frac{\ln 2}{\ln 3} = \log_3 2$

- (A) $\frac{\ln 3}{\ln 2}$ (B) $\log_3 2$ (C) $\log_3 5$ (D) $\log_7 2$ (E) 1

A 2. Solve for x : $2 \cdot 3^{x+1} = 5^{2x}$
 $\ln 2 + x \ln 3 + \ln 3 = 2x \ln 5$
 $x = \frac{\ln 2 + \ln 3}{2 \ln 5 - \ln 3} = \frac{\ln 6}{\ln 25 - \ln 3}$

- (A) $\frac{\ln 6}{\ln 25 - \ln 3}$ (B) $\frac{\ln 6}{\ln 25 - \ln 2}$ (C) $\frac{\ln 6}{\ln 25 - 1}$ (D) $\frac{\ln 6}{\ln 25 + \ln 3}$ (E) $\frac{\ln 6}{\ln 25 + \ln 2}$

D 3. Which of the following is equivalent to the function $f(x) = 9^x$?

- (A) $g(x) = \log_9 x$ (B) $g(x) = \log_9 9^x$ (C) $g(x) = x \ln 9$ (D) $g(x) = e^{(\ln 9)x}$ (E) $g(x) = -9^{-x}$

A 4. What is the domain of $y = -3 - 2 \ln(20 - x^2 - x)$?
 $-(x^2 + x - 20) > 0$
 $-(x+5)(x-4) > 0$
 $-5 < x < 4$

- (A) $(-5, 4)$ (B) $(-4, 5)$ (C) $(-\infty, -5) \cup (4, \infty)$ (D) $(-\infty, -4) \cup (5, \infty)$ (E) $(4, \infty)$

D 5. Which of the following is the inverse of $f(x) = 2 \cdot 3^x$?
 $x = 2 \cdot 3^y$
 $\frac{x}{2} = 3^y$
 $\log_3 \frac{x}{2} = y$

- (A) $f^{-1}(x) = 0.5 \log_3 x$ (B) $f^{-1}(x) = 3 \log_2 x$ (C) $f^{-1}(x) = \log_3 \left(\frac{x}{3} \right)$

- (D) $f^{-1}(x) = \log_3 \left(\frac{x}{2} \right)$ (E) $f^{-1}(x) = 2 \log_3 x$

C 6. If $\log_8(x+5) + \log_8 x = \log_8 24$, then x equals
 $x^2 + 5x - 24 = 0$
 $(x+8)(x-3) = 0$
 $x = -8, 3$

- (A) 0 (B) 8 (C) 3 (D) -5 (E) -8

E 7. Solve for x by getting the bases the same: $\left(\frac{1}{25} \right)^{x+1} \left(\sqrt{5^{40}} \right) = \left(\frac{125}{\sqrt{5}} \right)^{2x}$

- (A) $\frac{21}{9}$ (B) $\frac{18}{9}$ (C) $\frac{22}{3}$ (D) $\frac{21}{7}$ (E) $\frac{18}{7}$

B 8. $\log_{1/5} x^5 = \frac{5 \log_5 x}{\log_5 1/5} = -5 \log_5 x$

- (A) x^5 (B) $-5 \log_5 x$ (C) 5 (D) -5 (E) $5 \log_5 x$

E 9. If $B = \ln \left(\frac{1}{1-x} \right) + \ln \left(\frac{1}{1+x} \right)$, then $e^B =$

- (A) $\ln \left(\frac{1}{1-x^2} \right)$ (B) $\frac{2}{1-x^2}$ (C) $\frac{x}{1-x^2}$ (D) $\ln \left(\frac{x}{1-x^2} \right)$ (E) $\frac{1}{1-x^2}$

II. Free Response: **Show all work** in the space provided below the horizontal line. **Use correct units** where appropriate. **Give simplified, exact answers.**

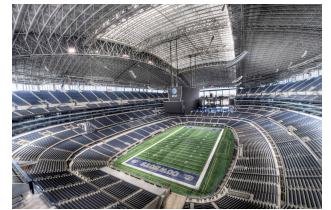
① B
② A
③ D
④ A
⑤ D
⑥ C
⑦ E
⑧ B
⑨ E

10. The decibel level, D , of sound is given by the equation

$$D = 10 \log \left(\frac{I}{I_0} \right) \quad (\text{equation 1})$$

where I is the intensity of the sound and I_0 is the lowest threshold of human sound.

- (a) Solve the above equation for I .
- (b) If $I_0 = 10^{-12}$ watts per square meter (**always**), and assuming the shout of a single person, in general, measures a maximum of 80 decibels, find the intensity, I , of an 80 decibel shout. Show the work that leads to your answer.
- (c) Using your Intensity value from part (b), if a group of 10 people attend a **math concert**, and each one of them shouts at an 80 decibel level, how loud, in decibels, will the sound be of all 10 of them shouting at the same time? That is, how loud would the sound be if the intensity, I , found in part (b) is increased by a factor of 10 and I is replaced in the original equation 1 with $10 \cdot I$?
- (d) AT&T stadium in Arlington, Texas (home of the Dallas Cowboys) has a seating capacity of 105,000 people. Assuming the upper pain threshold for human hearing is 130 decibels and a sellout crowd at a math concert held there, how many people in the crowd need to shout at the same time in order for the resulting sound level to meet or exceed this pain threshold? (Ignore any possible sound dampening or disappointment among Cowboy fans.) Show the work that leads to your answer.



(a) $D = 10 \log \left(\frac{I}{I_0} \right)$ ✓
 $\frac{D}{10} = \log_{10} \left(\frac{I}{I_0} \right)$ ✓
 $10^{D/10} = \frac{I}{I_0}$ ✓
 $I = I_0 \cdot 10^{D/10}$ ✓₂

(b) $I = 10^{-12} \cdot 10^{D/10}$ ✓
 $I(80) = 10^{-12} \cdot 10^{80/10}$ ✓₃
 $I(80) = 10^{-12} \cdot 10^8$
 $I(80) = 10^{-4} \text{ or } \frac{1}{10000}$ ✓₄

(c) Let $I = 10I$ ✓₅
 $D = 10 \log \left(\frac{10 \cdot 10^{-4}}{10^{-12}} \right)$ ✓
 $D = 10 \log (10^9)$ ✓
 $D = 90 \text{ dB}$ ✓₆ with or without units

(d) $I(130) = 10^{-12} \cdot 10^{130/10}$ ✓
 $I(130) = 10^{-12} \cdot 10^{13}$ ✓
 $I(130) = 10$ ✓
 So # people needed is $\frac{10^5}{10^{-4}}$ ✓
 $10^5 \text{ or } 100,000 \text{ ppl}$ ✓₇

7 total