Name KEY Date__________ Favorite Calculator Key_______

PreAP Precalculus

TEST Chapter 3.1-3.2 Form A. No Calculator Permitted

Part I: Multiple Choice—Put your CAPITAL LETTER answer choice in the blank to the left of the number.

B 1. Let *R, S, T,* and *V* be the roots of $f(x)=2x^4-3x^3-24x^2+13x+12$. If $f\left(-\frac{1}{2}\right)=0$ and if $(x-1)$ is a factor of $f(x)$, find the product *RSTV*.

(A) –6 (B) 6 (C) 12 (D) –12 (E) 3

	2	−3	−24	13	12
1	↓	2	−1	−25	−12
	2	−1	−25	−12	0
$-\frac{1}{2}$	↓	−1	1	12	
	2	−2	−24	0	

$2x^2-2x-24=0$

$2(x^2-x-12)=0$

$2(x-4)(x+3)=0$

$x=4, x=-3$

So roots are $x=-\frac{1}{2}, 1, 4, -3$

Their product is $\left(-\frac{1}{2}\right)(1)(4)(-3)$

6

C 2. Which of the following MUST be true about a polynomial function of odd degree?

(A) It is an odd function (B) It has the same end behaviors (C) It has at least one real root

(D) It has an even number of irrational roots (E) It has an odd number of relative extrema

E 3. A linear factor of $2x^3-6x^2-48x+160$ is $(x+5)$ and what other factor?

(A) $x+2$ (B) $x+3$ (C) $x-2$ (D) $x-3$ (E) $x-4$

	2	−6	−48	160
−5	↓	−10	80	−160
	2	−16	32	0

$2(x^2-8x+16)=0$

$2(x-4)^2=0$

A 4. The value of *k* that will make $(x-1)$ a factor of $g(x)=kx^3-17x^2-4kx+8$ is:

(A) –3 (B) 3 (C) –5 (D) 4 (E) -1

$g(1)=k-17-4k+8=0$

$-3k=9$

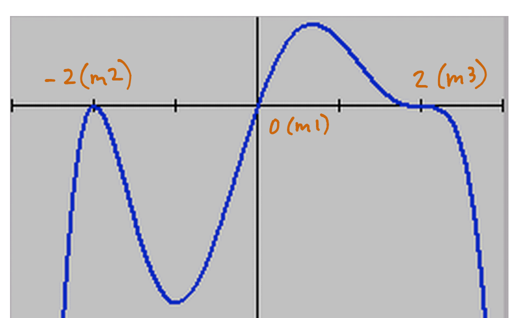
$k=-3$

D 5. Which of the specified functions might have the given graph?

(A) $f(x)=-x(x+2)^2(2-x)^3$ (B) $f(x)=-x^2(x+2)(x-2)^3$

(C) $f(x)=x(x+2)^2(x-2)^3$ (D) $f(x)=x(x+2)^2(2-x)^3$

(E) $f(x)=-x(x+2)^3(x-2)^2$



$y=-x(x+2)^2(x-2)^3$

OR $y=x(x+2)^2(2-x)^3$

B ______ 6. If a polynomial function with rational coefficients of degree 7 has roots of 0, –1, 2, $\sqrt{2}+1$ and $4-6\sqrt{5}$, then another root must be:

$-\sqrt{2}+1$ or $1-\sqrt{2}$

(A) $\sqrt{2}-1$ (B) $1-\sqrt{2}$ (C) $-4-6\sqrt{5}$ (D) $-\sqrt{2}-1$ (E) $-4+6\sqrt{5}$

$4+6\sqrt{5}$

A ______ 7. What is the remainder when $f(x)=4(x-2)^{33}-7$ is divided by $(x-1)$?

(A) -11 (B) -3 (C) 3 (D) 11 (E) -15

$f(1) = 4(-1)^{33}-7$
$= -4-7$
$= -11$

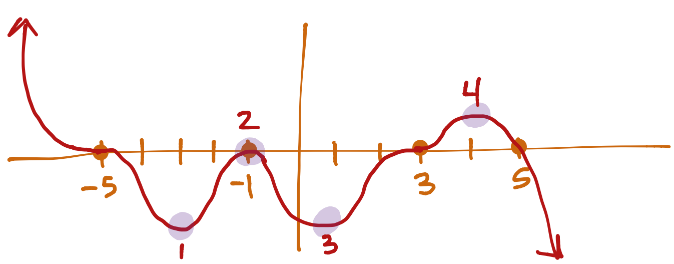
D ______ 8. An equation of a polynomial of the form $y = A\cdot f(x)$ of lowest degree with rational coefficients and the following characteristics $f(2) = f(-3) = f(\sqrt{3}) = 0$ and $f(1) = 4$, has a vertical dilation value of $A =$

(A) 4 (B) $-\frac{2}{3}$ (C) -2 (D) $\frac{1}{2}$ (E) 2

$f(x) = A(x-2)(x+3)(x-\sqrt{3})(x+\sqrt{3})$
$f(x) = A(x-2)(x+3)(x^2-3)$
@ (1,4): $4 = A(-1)(4)(-2)$
$4 = A\cdot 8$
$A = \frac{1}{2}$

A ______ 9. An equation of an 9[th] degree polynomial with a negative leading coefficient whose only roots are $x=-5(m3)$, $x=-1(m2)$, $x=3(m3)$ and $x=5(m1)$ has exactly how many relative extrema?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8



4 Local Extrema

Part II: Free Response—Show all work in the space provided. Use proper notation.

10. For $f(x) = 30 + 56x - 25x^2 - 32x^3 + 5x^4 + 2x^5$

(a) Find the range of $f(x)$.

$f(x) = 2x^5 + 5x^4 - 32x^3 - 25x^2 + 56x + 30$

$R_f : \mathbb{R}$ or $(-\infty, \infty)$ or $\{y \mid y \in \mathbb{R}\}$ $\sqrt{1}$

(b) What is the coordinate, (x, y), of the y-intercept?

$f(0) = 30$

$(0, 30)$ $\sqrt{2}$

(c) List the distinct, possible, rational roots of $f(x)$. According to the Rational Root Theorem, how many distinct possible roots of $f(x)$ are there?

$30: \pm 1, \pm 30, \pm 2, \pm 15, \pm 3, \pm 10, \pm 5, \pm 6,$
$2: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 30, \pm 2, \pm 15, \pm 3, \pm 10, \pm 5, \pm 6,$
$\pm \frac{1}{2}, \pm \cancel{\frac{30}{2}}, \pm \cancel{\frac{2}{2}}, \pm \frac{15}{2}, \pm \frac{3}{2}, \pm \cancel{\frac{10}{2}}, \pm \frac{5}{2}, \pm \cancel{\frac{6}{2}}$

So, there are $12 \times 2 (\pm)$ or 24 $\sqrt{3}$ distinct possible rational roots.

* Correct answer must be in the presence of all 24.

(d) Determine both the left and right end behaviors. Use proper limit notation.

$\lim_{x \to \infty} f(x) = \infty$ $\sqrt{4}$

$\lim_{x \to -\infty} f(x) = -\infty$ $\sqrt{5}$

(e) Given that $f\left(-\sqrt{2}\right)=0$ and $(x+5)$ is a factor of $f(x)$, use (and show) synthetic division to find the roots of $f(x)$. List ALL the roots of $f(x)$ at the end as $x =$

$f(-\sqrt{2})=0, f(\sqrt{2})=0, f(-5)=0$

$$
\begin{array}{r|rrrrrr}
 & 2 & 5 & -32 & -25 & 56 & 30 \\
-5 & \downarrow & -10 & 25 & 35 & -50 & -30 \\
\hline
 & 2 & -5 & -7 & 10 & 6 & \boxed{0} \quad ✓6 \\
\sqrt{2} & \downarrow & 2\sqrt{2} & 4-5\sqrt{2} & -3\sqrt{2}-10 & -6 & \\
\hline
 & 2 & 2\sqrt{2}-5 & -3-5\sqrt{2} & -3\sqrt{2} & \boxed{0} & \\
-\sqrt{2} & \downarrow & -2\sqrt{2} & 5\sqrt{2} & 3\sqrt{2} & & \\
\hline
 & 2 & -5 & -3 & \boxed{0} & &
\end{array}
$$

So, $2x^2-5x-3=0$

$(2x+1)(x-3)=0$

So, f has roots of

$x = \underbrace{-5, -\sqrt{2}}_{✓7}, \ \sqrt{2}\ (✓8), \ -\frac{1}{2}\ (✓9), \ 3\ (✓10)$