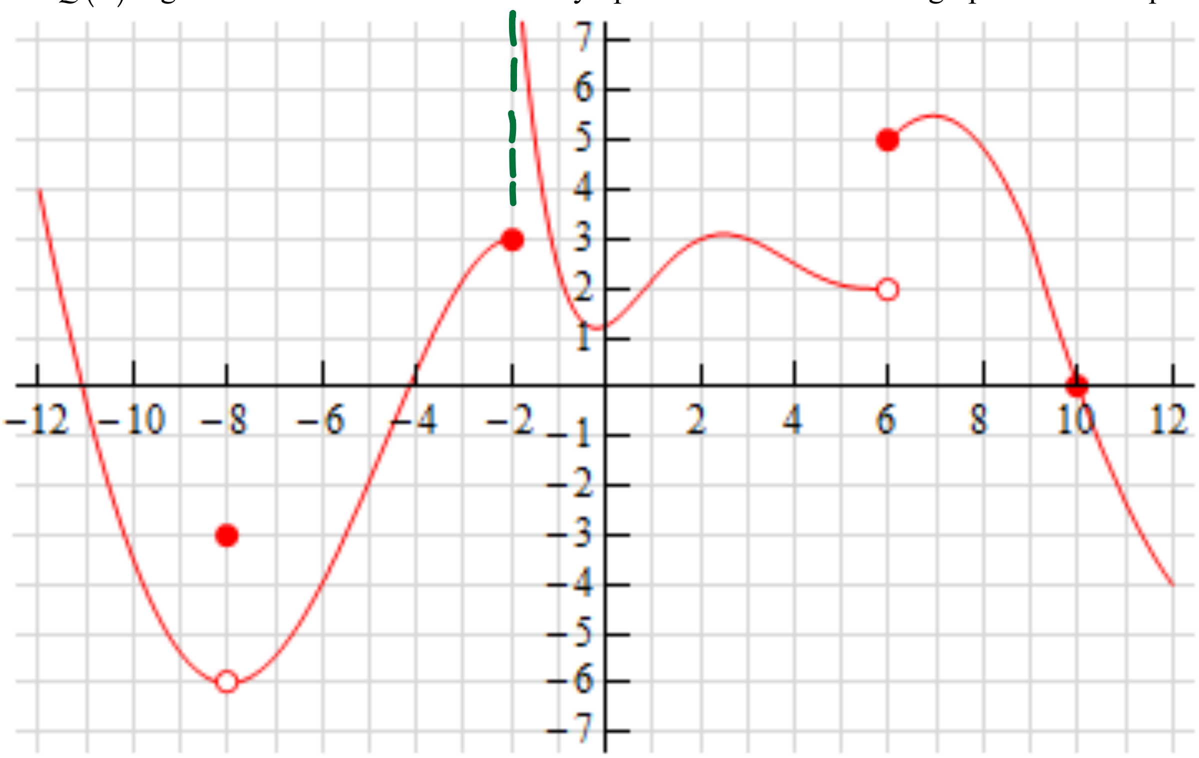
Name *KEY* Date *Mon, Sept 24, 2018* Period *.*

PCPAP TEST: Chapter 1.1-2.2 2018-2019
No Calculator, Form A *18 checks total*

(A) B E B B E B D D D | (B) B B E B B D D D E | (C) B B B E D D D E B

Part I: Multiple Choice. Put the CAPITAL letter in each blank to the left of the problem number.

The graph of $Q(x)$ is give below. It has a Vertical Asymptote at $x = -2$. Use the graph to answer questions 1-4.



B 1. $Q(10) =$ (A) 10 (B) 0 (C) -3 (D) -4 (E) DNE

E 2. $\lim\limits_{x \to 6} Q(x) =$ (A) 2 (B) 5 (C) 0 (D) 4 (E) DNE

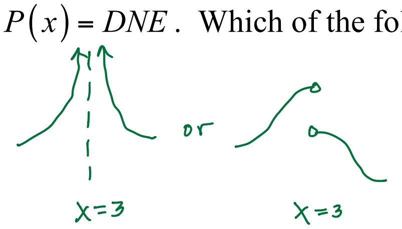
$\lim\limits_{x \to 6^-} Q(x) = 2 \neq \lim\limits_{x \to 6^+} Q(x) = 5$

B 3. $\lim\limits_{x \to -2^+} Q(x) =$ (A) $-\infty$ (B) ∞ (C) 3 (D) 7 (E) 0

B 4. $\lim\limits_{x \to -8^-} Q(x) =$ (A) -3 (B) -6 (C) 0 (D) -4 (E) DNE

E 5. A function $P(x)$ has the property that $\lim\limits_{x \to 3} P(x) = DNE$. Which of the following discontinuities can the graph of $P(x)$ possibly have at $x = 3$?

I. Removable Point Discontinuity
II. Non-Removable Jump Discontinuity
III. Non-Removable Infinite Discontinuity



(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

B 6. Simplify: $\dfrac{2x^{-1}y^2-3x^{-2}y^{-1}}{4x^{-3}y^3+5x^4y}$

(A) $\dfrac{x^2y^3-3x}{2y^4+5x^{12}y^2}$ (B) $\dfrac{2x^2y^3-3x}{4y^4+5x^7y^2}$ (C) $\dfrac{2x^2y^3-3x}{4y^4+5x^{12}y^2}$ (D) $\dfrac{x^2y^3-3x}{2y^4+5x^7y^2}$ (E) $\dfrac{2x^2-3}{4y+5x^6y^2}$

$$\frac{\frac{2y^2}{x}-\frac{3}{x^2y}}{\frac{4y^3}{x^3}+\frac{5x^4y}{1}}\cdot\left(\frac{\frac{x^3y}{1}}{\frac{x^3y}{1}}\right) \leftarrow \text{LCM}$$

$$\frac{2x^2y^3-3x}{4y^4+5x^7y^2}$$

D 7. If $f(x)=\begin{cases}-2x-8, & x<-3\\ 4, & x=-3\\ x^2-11, & x>-3\end{cases}$. Which of the following is NOT true about $f(x)$?

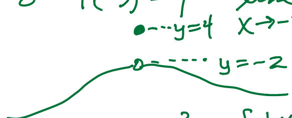
(A) $f(-4)=0$ (B) $\lim\limits_{x\to-3^-}f(x)=-2$ (C) $\lim\limits_{x\to-3^+}f(x)=-2$

(D) $f(x)$ has a jump discontinuity at $x=-3$ (E) $D_f:(-\infty,\infty)$

$\lim\limits_{x\to-3^-}f(x)=-2(-3)-8 = 6-8 = -2$

$f(-3)=4$

$\lim\limits_{x\to-3^+}f(x)=(-3)^2-11 = 9-11 = -2$



$x=-3$ f has a hole at $(-3,-2)$

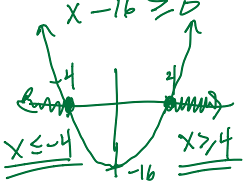
$f(-4)=(-2)(-4)-8 = 8-8 = 0$

D 8. Find the domain of $h(x)=\dfrac{\sqrt{x^2-16}}{\sqrt{x}+2}$. D_h:

(A) $\{x|x\le-4 \text{ or } x\ge4\}$ (B) $\{x|0<x<4\}$ (C) $\{x|x\le-4 \text{ or } x>4\}$ (D) $\{x|x\ge4\}$ (E) $\{x|x>4\}$

Rad I

$x^2-16\ge0$



$x\le-4$ $x\ge4$

Rad II

$x\ge0$

Denom

$\sqrt{x}+2\ne0$

$\sqrt{x}\ne-2$

$x\ne4$

but $\sqrt{4}+2\ne0$

so $x=4$ is extraneous

So, $x\ge4$, since when $x\ge0$, $x\le-4$ and $x\ge4$

D 9. The function $f(x)=\dfrac{x^2+2x-8}{x^2+10x+24}$ has a removable point discontinuity (hole) at

(A) $\left(4,-\frac{1}{5}\right)$ (B) $\left(-4,-\frac{1}{5}\right)$ (C) $\left(-4,\frac{1}{5}\right)$ (D) $(-4,-3)$ (E) $(-4,3)$

$$f(x)=\frac{\cancel{(x+4)}(x-2)}{\cancel{(x+4)}(x+6)}$$

hole at $\left(-4,\frac{-4-2}{-4+6}\right)$

$\left(-4,\frac{-6}{2}\right)$

$(-4,-3)$

Part II: Free Response

Show all work in a logical, vertical sequence. Remember that your process and notation are worth points too, not just your answer. You must communicate your results properly, not just attain them.

10. For the following functions,

$f(x)=\sqrt{x+5}-4$, $g(x)=\sqrt{-x+9}$, $h(x)=x^2-2x-15$ answer the following questions.

(a) Set up and simplify the **equation** for the function $K(x)=g(h(x))$, and then find the domain of $K(x)$ by sketching a graph of your radicand. **Be careful to use parentheses when substituting in for x, so that you may distribute any necessary values** Show the work, including the graph, that leads to your answer. Give your domain in either proper set or interval notation.

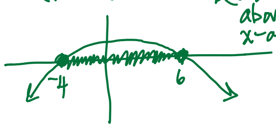
$$K(x)=g(h(x))$$
$$=\sqrt{-(x^2-2x-15)+9}$$
$$=\sqrt{-x^2+2x+15+9}$$
$$=\sqrt{-x^2+2x+24}$$

Domain

$-x^2+2x+24\geq 0$

$-(x^2-2x-24)\geq 0$

$-(x-6)(x+4)\geq 0$ on or above x-axis



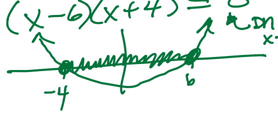
or

Domain

$-x^2+2x+24\geq 0$

$x^2-2x-24\leq 0$

$(x-6)(x+4)\leq 0$ on or below x-axis



√1 graph

$D_K:\{x\,|\,-4\leq x\leq 6\}$ or $[-4,6]$ √2

(b) Set up the **equation** for the function $R(x)=\dfrac{2x-10}{h(x)}$. Determine the equation and/or coordinates (x,y), of any vertical asymptotes and holes, respectively. Show the work that leads to your answer, and use proper notation.

$$R(x)=\frac{2x-10}{x^2-2x-15}$$
$$=\frac{2(x-5)}{(x-5)(x+3)}$$
$$=\frac{2}{x+3},\ x\neq 5$$

R has a VA @ $x=-3$ √3

hole, x-value √4

R has a hole @ $(5,\frac{2}{8})=(5,\frac{1}{4})$ √5 y-value

(c) Set up the **equation** for the function $J(x) = \dfrac{h(x)}{f(x)}$, and then find the domain of $J(x)$. Show the work that leads to your answer. Give your domain in either proper set or interval notation.

$$J(x) = \frac{x^2 - 2x - 15}{\sqrt{x+5} - 4}$$

Rad

$x + 5 \geq 0$

$x \geq -5$

Denom

$\sqrt{x+5} - 4 \neq 0$

$\sqrt{x+5} \neq 4$

$x + 5 \neq 16$

$x \neq 11$

check: $\sqrt{11+5} - 4$

$\sqrt{16} - 4$

$4 - 4$

0 ✓ not extraneous, does cause zero

✓6 ✓7

$D_J: \{x \mid x \geq -5, x \neq 11\}$

or

$[-5, 11) \cup (11, \infty)$

(d) Set up and **completely simplify** $\dfrac{h(x+w) - h(x)}{w}$ for some constant w. Show the work that leads to your answer.

$h(x) = x^2 - 2x - 15$

$$\frac{(x+w)^2 - 2(x+w) - 15 - (x^2 - 2x - 15)}{w}$$ ✓8 setup & process

or $$\frac{(x+w)^2 - 2(x+w) - 15 - x^2 + 2x + 15}{w}$$

$$\frac{\cancel{x^2} + 2xw + w^2 \cancel{- 2x} - 2w \cancel{- 15} \cancel{- x^2} \cancel{+ 2x} \cancel{+ 15}}{w}$$

$$\frac{2xw + w^2 - 2w}{w}$$

$$\frac{\cancel{w}(2x + w - 2)}{\cancel{w}}$$

$$2x + w - 2$$ ✓9

9 checks