

Name \_\_\_\_\_ Date \_\_\_\_\_ Period \_\_\_\_\_

TEST Taylor Polynomials and Taylor Series

Calculator Permitted

**Multiple Choice**

\_\_\_\_ 1. The Taylor series for  $\sin x$  about  $x=0$  is  $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ . If  $f$  is a function such that

$f'(x) = \sin(x^2)$ , then the coefficient of  $x^7$  in the Taylor series for  $f(x)$  about  $x=0$  is

- (A)  $\frac{1}{7!}$       (B)  $\frac{1}{7}$       (C) 0      (D)  $-\frac{1}{42}$       (E)  $-\frac{1}{7!}$

\_\_\_\_ 2. What are all values of  $x$  for which the series  $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$  converges?

- (A)  $-3 \leq x \leq 3$       (B)  $-3 < x < 3$       (C)  $-1 < x \leq 5$       (D)  $-1 \leq x \leq 5$       (E)  $-1 \leq x < 5$

\_\_\_\_ 3. Let  $f$  be the following function given by  $f(x) = \ln(3-x)$ . The third-degree Taylor polynomial for  $f$  about  $x=2$  is

- (A)  $-(x-2) + \frac{(x-2)^2}{2} - \frac{(x-2)^3}{3}$       (B)  $-(x-2) - \frac{(x-2)^2}{2} - \frac{(x-2)^3}{3}$       (C)  $(x-2) + (x-2)^2 + (x-2)^3$   
(D)  $(x-2) + \frac{(x-2)^2}{2} + \frac{(x-2)^3}{3}$       (E)  $(x-2) - \frac{(x-2)^2}{2} + \frac{(x-2)^3}{3}$

\_\_\_\_ 4. If  $f(x) = \sum_{k=1}^{\infty} (\sin^2 x)^k$ , then  $f(1)$  is

- (A) 0.369      (B) 0.585      (C) 2.400      (D) 2.426      (E) 3.426

\_\_\_\_ 5. A function  $f$  has Maclaurin series given by  $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots + \frac{x^{2n}}{(2n)!} + \dots$ . Which of the

following is an expression for  $f(x)$ ?

- (A)  $\cos x$       (B)  $e^x - \sin x$       (C)  $e^x + \sin x$       (D)  $\frac{1}{2}(e^x + e^{-x})$       (E)  $e^{x^2}$

## Free Response

(2002-BC6) The Maclaurin series for the function  $f$  is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1} = 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4} + \cdots + \frac{(2x)^{n+1}}{n+1} + \cdots$$

on its interval of convergence.

- (a) Find the interval of convergence of the Maclaurin series for  $f$ . Justify your answer.
  - (b) Find the first four terms and the general term for the Maclaurin series for  $f'(x)$ .
  - (c) Use the Maclaurin series you found in part (b) to find the value of  $f'\left(-\frac{1}{3}\right)$ .
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