Name____________________________________ Date______________________ Period__________

TEST Taylor Polynomials and Taylor Series

Calculator Permitted

Multiple Choice

_____ 1. The Taylor series for $\sin x$ about $x=0$ is $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$. If *f* is a function such that $f'(x)=\sin\left(x^2\right)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x=0$ is

(A) $\frac{1}{7!}$ (B) $\frac{1}{7}$ (C) 0 (D) $-\frac{1}{42}$ (E) $-\frac{1}{7!}$

_____ 2. What are all values of *x* for which the series $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n\cdot 3^n}$ converges?

(A) $-3\le x\le 3$ (B) $-3<x<3$ (C) $-1<x\le 5$ (D) $-1\le x\le 5$ (E) $-1\le x<5$

_____ 3. Let *f* be the following function given by $f(x)=\ln(3-x)$. The third-degree Taylor polynomial for *f* about $x=2$ is

(A) $-(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$ (B) $-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$ (C) $(x-2)+(x-2)^2+(x-2)^3$

(D) $(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$ (E) $(x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

_____ 4. If $f(x)=\sum_{k=1}^{\infty}\left(\sin^2 x\right)^k$, then $f(1)$ is

(A) 0.369 (B) 0.585 (C) 2.400 (D) 2.426 (E) 3.426

_____ 5. A function *f* has Maclaurin series given by $1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots+\frac{x^{2n}}{(2n)!}+\cdots$. Which of the following is an expression for $f(x)$?

(A) $\cos x$ (B) $e^x-\sin x$ (C) $e^x+\sin x$ (D) $\frac{1}{2}\left(e^x+e^{-x}\right)$ (E) e^{x^2}

Free Response

(2002-BC6) The Maclaurin series for the function *f* is given by

$$f(x)=\sum_{n=0}^{\infty}\frac{(2x)^{n+1}}{n+1}=2x+\frac{4x^2}{2}+\frac{8x^3}{3}+\frac{16x^4}{4}+\cdots+\frac{(2x)^{n+1}}{n+1}+\cdots$$

on its interval of convergence.

(a) Find the interval of convergence of the Maclaurin series for *f*. Justify your answer.

(b) Find the first four terms and the general term for the Maclaurin series for $f'(x)$.

(c) Use the Maclaurin series you found in part (b) to find the value of $f'\left(-\frac{1}{3}\right)$.