Name_____ Date_____ Period_____

TEST BC CH 8.1-9.1

No Calculator (except on F.R., #13 & #14)

I. Multiple Choice: Put the capital letter of the correct answer in the blank.

_____ 1. At time $t \geq 0$, a particle moving in the *xy*-plane has velocity vector given by $v(t) = \langle t^2, 5t \rangle$. What is the acceleration vector of the particle at time $t = 3$?

(A) $\left\langle 9, \frac{45}{2} \right\rangle$ (B) $\langle 6,5 \rangle$ (C) $\langle 2,0 \rangle$ (D) $\sqrt{306}$ (E) $\sqrt{61}$

_____ 2. Consider the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

(A) $\lim_{n\to\infty} \frac{e}{n!} < 1$ (B) $\lim_{n\to\infty} \frac{n!}{e} < 1$ (C) $\lim_{n\to\infty} \frac{n+1}{e} < 1$ (D) $\lim_{n\to\infty} \frac{e}{n+1} < 1$ (E) $\lim_{n\to\infty} \frac{e}{(n+1)!} < 1$

_____ 3. Which of the following gives the length of the path described by the parametric equations $x = \sin t^3$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$?

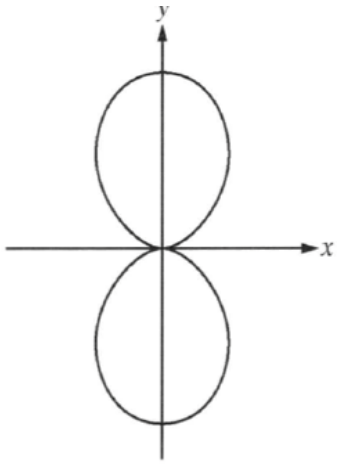
(A) $\int_0^{\pi} \sqrt{\sin^2(t^3) + e^{10t}}\, dt$ (B) $\int_0^{\pi} \sqrt{\cos^2(t^3) + e^{10t}}\, dt$ (C) $\int_0^{\pi} \sqrt{9t^4 \cos^2(t^3) + 25e^{10t}}\, dt$

(D) $\int_0^{\pi} \sqrt{3t^2 \cos^2(t^3) + 5e^{10t}}\, dt$ (E) $\int_0^{\pi} \sqrt{\cos^2(3t^2) + e^{10t}}\, dt$

_____ 4. Let *R* be the region between the graph of $y = e^{-2x}$ and the *x*-axis for $x \geq 3$. The area of *R* is

(A) $\frac{1}{2e^6}$ (B) $\frac{1}{e^6}$ (C) $\frac{2}{e^6}$ (D) $\frac{\pi}{2e^6}$ (E) infinite

_____ 5. Which of the following expressions gives the total area enclosed by the polar curve $r = \sin^2\theta$ shown in the figure?



(A) $\frac{1}{2}\int_0^{\pi} \sin^2\theta\, d\theta$ (B) $\int_0^{\pi} \sin^2\theta\, d\theta$ (C) $\frac{1}{2}\int_0^{\pi} \sin^4\theta\, d\theta$

(D) $\int_0^{\pi} \sin^4\theta\, d\theta$ (E) $2\int_0^{\pi} \sin^4\theta\, d\theta$

_____ 6. Let *f* be a positive, continuous, decreasing function such that $a_n = f(n)$. If $\sum_{n=1}^{\infty} a_n$ converges to *k*, which of the following must be true?

(A) $\lim_{n\to\infty} a_n = k$ (B) $\int_1^n f(x)\,dx = k$ (C) $\int_1^{\infty} f(x)\,dx$ diverges (D) $\int_1^{\infty} f(x)\,dx$ converges (E) $\int_1^{\infty} f(x)\,dx = k$

_____ 7. If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \le a_n \le b_n$ for all n, which of the following statements must be true?

(A) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges (B) $\sum_{n=1}^{\infty} (-1)^n b_n$ converges (C) $\sum_{n=1}^{\infty} (-1)^n b_n$ diverges

(D) $\sum_{n=1}^{\infty} b_n$ converges (E) $\sum_{n=1}^{\infty} b_n$ diverges

_____ 8. What are all values of p for which $\int_1^{\infty} \frac{1}{x^{2p}}\,dx$ converges?

(A) $p < -1$ (B) $p > 0$ (C) $p > \frac{1}{2}$ (D) $p > 1$

(E) There are no values of p for which this integral converges.

_____ 9. The position of a particle moving in the xy-plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest?

(A) -1 only (B) 0 only (C) 2 only (D) -1 and 2 only (E) -1 , 0, and 2

_____ 10. What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

(A) 1 (B) 2 (C) 4 (D) 6 (E) The series diverges

_____ 11. What are all values of p for which the infinite series $\sum_{n=1}^{\infty} \frac{n}{n^p+1}$ converges?

(A) $p > 0$ (B) $p \geq 1$ (C) $p > 1$ (D) $p \geq 2$ (E) $p > 2$

_____ 12. Which of the following series diverge?

I. $\sum_{n=0}^{\infty} \left(\frac{\sin 2}{\pi} \right)^n$ II. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$ III. $\sum_{n=1}^{\infty} \left(\frac{e^n}{e^n+1} \right)$

(A) III only (B) I and II only (C) I and III only (D) II and III only (E) I, II, and III

13. For $t \geq 0$, a particle is moving along a curve so that its position at time t is $\left(x(t), y(t)\right)$. At time $t = 2$, the particle is at position $(1,5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

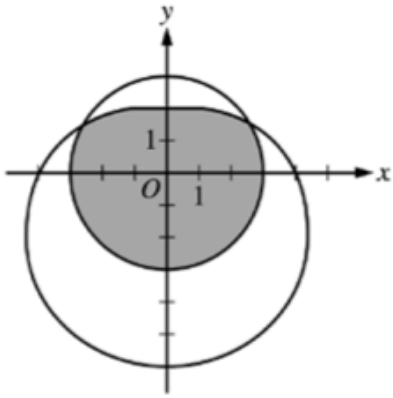
(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$. Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

14. (Calculator Permitted) The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure at right. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.



(a) Let *S* be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of *S*.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time *t* seconds, $\theta = t^2$. Find the time *t* in the interval $1 \le t \le 2$ for which the *x*-coordinate of the particle's position is -1.

(c) For the particle described in part (b), find the position vector in terms of *t*. Find the velocity vector at time $t = 1.5$.