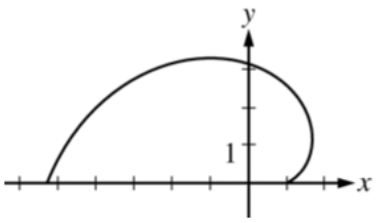
Name_____ Date_____ Period_____
TEST BC CH 7.1-8.2
Calculator Permitted

I. Multiple Choice: Put the capital letter of the correct answer in the blank.

_____ 1. Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

(A) $3\int_0^{\pi/2} \cos^2\theta d\theta$ (B) $3\int_0^{\pi} \cos^2\theta d\theta$ (C) $\frac{3}{2}\int_0^{\pi/2} \cos^2\theta d\theta$ (D) $3\int_0^{\pi/2} \cos\theta d\theta$ (E) $3\int_0^{\pi} \cos\theta d\theta$




_____ 2. The graph above shows the polar curve $r = 2\theta + \cos\theta$ for $0 \le \theta \le \pi$. What is the area of the region bounded by the curve and the *x*-axis?

(A) 3.069 (B) 4.935 (C) 9.870 (D) 17.456 (E) 34.912

_____ 3. A particle moves in the *xy*-plane so that its position at any time *t* is given by $x(t) = t^2$ and $y(t) = \sin(4t)$. What is the speed of the particle when $t = 3$?

(A) 2.909 (B) 3.062 (C) 6.884 (D) 9.016 (E) 47.393

_____ 4. At time $t \geq 0$, a particle moving in the *xy*-plane has velocity vector given by $\vec{v}(t) = \langle t^2, 5t \rangle$. What is the acceleration vector of the particle at time $t = 3$?

(A) $\left\langle 9, \frac{45}{2} \right\rangle$ (B) $\langle 6,5 \rangle$ (C) $\langle 2,0 \rangle$ (D) $\sqrt{306}$ (E) $\sqrt{61}$

_____ 5. Which of the following gives the length of the path described by the parametric equations $x = \sin t^3$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$ ?

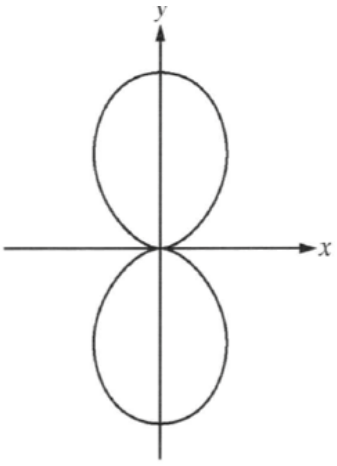
(A) $\int_0^{\pi} \sqrt{\sin^2\left(t^3\right) + e^{10t}}\, dt$ (B) $\int_0^{\pi} \sqrt{\cos^2\left(t^3\right) + e^{10t}}\, dt$ (C) $\int_0^{\pi} \sqrt{9t^4 \cos^2\left(t^3\right) + 25e^{10t}}\, dt$

(D) $\int_0^{\pi} \sqrt{3t^2 \cos^2\left(t^3\right) + 5e^{10t}}\, dt$ (E) $\int_0^{\pi} \sqrt{\cos^2\left(3t^2\right) + e^{10t}}\, dt$

_____ 6. Which of the following expressions gives the total area enclosed by the polar curve $r = \sin^2\theta$ shown in the figure?

(A) $\frac{1}{2}\int_0^{\pi} \sin^2\theta\, d\theta$ (B) $\int_0^{\pi} \sin^2\theta\, d\theta$ (C) $\frac{1}{2}\int_0^{\pi} \sin^4\theta\, d\theta$

(D) $\int_0^{\pi} \sin^4\theta\, d\theta$ (E) $2\int_0^{\pi} \sin^4\theta\, d\theta$



_____ 7. The position of a particle moving in the *xy*-plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$ . For what values of *t* is the particle at rest?

(A) $-1$ only (B) 0 only (C) 2 only (D) $-1$ and 2 only (E) $-1$ , 0, and 2

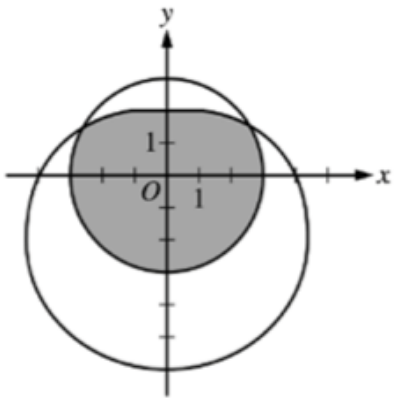
_____ 8. What is $\frac{dy}{dx}$ for $r = 6\cos 4\theta$ ?

(A) $-\frac{\cos 4\theta \cos\theta - \sin 4\theta \sin\theta}{\cos 4\theta \sin\theta + \sin 4\theta \cos\theta}$ (B) $\frac{\cos 4\theta \cos\theta - 4\sin 4\theta \sin\theta}{\cos 4\theta \sin\theta + 4\sin 4\theta \cos\theta}$ (C) $-\frac{\cos 4\theta \cos\theta}{\cos 4\theta \sin\theta + 4\sin 4\theta \cos\theta}$

(D) $-\frac{\cos 4\theta \cos\theta - 4\sin 4\theta \sin\theta}{\cos 4\theta \sin\theta}$ (E) $-\frac{\cos 4\theta \cos\theta - 4\sin 4\theta \sin\theta}{\cos 4\theta \sin\theta + 4\sin 4\theta \cos\theta}$

_____ 9. If $x(t) = \cos(2t)$ and $y(t) = \sin(2t)$, which of the following is equal to $\frac{d^2y}{dx^2}$?

(A) $2\csc^2(2t)$ (B) $-\csc^3(2t)$ (C) $\csc^3(2t)$ (D) $-2\csc^3(2t)$ (E) $-2\csc^2(2t)$




14. The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(a) Let *S* be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of *S*.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time *t* seconds, $\theta = t^2$. Find the time *t* in the interval $1 \le t \le 2$ for which the *x*-coordinate of the particle's position is $-1$.

(c) For the particle described in part (b), find the position vector in terms of *t*. Find the velocity vector at time $t = 1.5$.