Name____________________________________ Date______________________ Period__________________

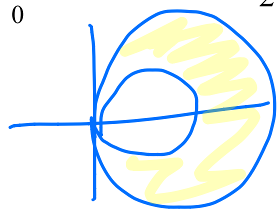
TEST BC CH 9.1-10.2

Calculator Permitted

I. Multiple Choice: Put the capital letter of the correct answer in the blank.

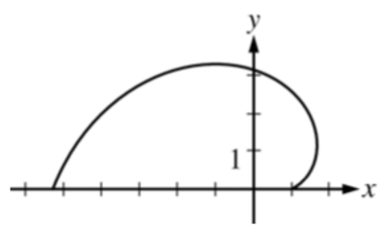
A 1. Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

(A) $3\int_0^{\pi/2} \cos^2\theta\, d\theta$ (B) $3\int_0^{\pi} \cos^2\theta\, d\theta$ (C) $\frac{3}{2}\int_0^{\pi/2} \cos^2\theta\, d\theta$ (D) $3\int_0^{\pi/2} \cos\theta\, d\theta$ (E) $3\int_0^{\pi} \cos\theta\, d\theta$



$\frac{1}{2}\int_0^{\pi}\left[(2\cos\theta)^2 - \cos^2\theta\right]d\theta$

$\frac{3}{2}\int_0^{\pi} \cos^2$ or $3\int_0^{\pi/2}$




D 2. The graph above shows the polar curve $r = 2\theta + \cos\theta$ for $0 \le \theta \le \pi$. What is the area of the region bounded by the curve and the $x$-axis?

(A) 3.069 (B) 4.935 (C) 9.870 (D) 17.456 (E) 34.912

$A = \frac{1}{2}\int_0^{\pi} (2\theta + \cos\theta)^2\, d\theta$

17.4562

C 3. A particle moves in the $xy$-plane so that its position at any time $t$ is given by $x(t) = t^2$ and $y(t) = \sin(4t)$. What is the speed of the particle when $t = 3$?

(A) 2.909 (B) 3.062 (C) 6.884 (D) 9.016 (E) 47.393

$x' = 2t$ $y' = 4\cos 4t$

$x'(3) = 6$ $y'(3) = 4\cos 12$

Speed $= \sqrt{36 + 16(\cos 12)^2}$

6.8842

B ______ 4. At time $t \geq 0$, a particle moving in the $xy$-plane has velocity vector given by $\vec{v}(t) = \langle t^2, 5t \rangle$. What is the acceleration vector of the particle at time $t = 3$?

(A) $\left\langle 9, \frac{45}{2} \right\rangle$ (B) $\langle 6,5 \rangle$ (C) $\langle 2,0 \rangle$ (D) $\sqrt{306}$ (E) $\sqrt{61}$

$\vec{v}\,'(t) = \vec{a}(t) = \langle 2t, 5 \rangle$

$\vec{a}(3) = \langle 6, 5 \rangle$

C ______ 5. Which of the following gives the length of the path described by the parametric equations $x = \sin t^3$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$ ?

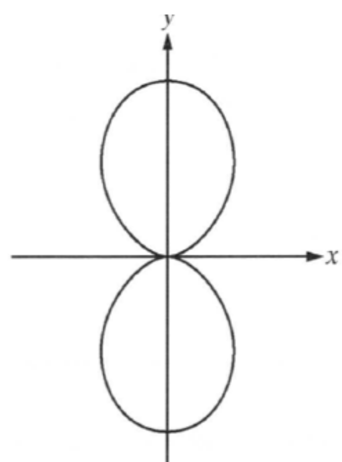
(A) $\int_0^\pi \sqrt{\sin^2(t^3) + e^{10t}}\, dt$ (B) $\int_0^\pi \sqrt{\cos^2(t^3) + e^{10t}}\, dt$ (C) $\int_0^\pi \sqrt{9t^4 \cos^2(t^3) + 25e^{10t}}\, dt$

(D) $\int_0^\pi \sqrt{3t^2 \cos^2(t^3) + 5e^{10t}}\, dt$ (E) $\int_0^\pi \sqrt{\cos^2(3t^2) + e^{10t}}\, dt$

$x' = 3t^2 \cos t^3 \quad y' = 5e^{5t}$

$L = \int_0^\pi \sqrt{9t^4 \cos^2 t^3 + 25e^{10t}}$

D ______ 6. Which of the following expressions gives the total area enclosed by the polar curve $r = \sin^2\theta$ shown in the figure?




(A) $\frac{1}{2}\int_0^\pi \sin^2\theta\, d\theta$ (B) $\int_0^\pi \sin^2\theta\, d\theta$ (C) $\frac{1}{2}\int_0^\pi \sin^4\theta\, d\theta$

(D) $\int_0^\pi \sin^4\theta\, d\theta$ (E) $2\int_0^\pi \sin^4\theta\, d\theta$

$\sin^2\theta = 0$

$\sin\theta = 0$

$\theta = 0, \pi$

Area $= \frac{1}{2}\int_0^\pi \sin^4\theta\, d\theta$

C
______ 7. The position of a particle moving in the $xy$-plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$ . For what values of $t$ is the particle at rest?

(A) $-1$ only (B) 0 only (C) 2 only (D) $-1$ and 2 only (E) $-1$ , 0, and 2

$x' = 3t^2 - 6t = 0$
$3t(t-2)$
$t=0, t=2$

$y' = 6t^2 - 6t - 12 = 0$
$6(t^2 - t - 2) = 0$
$6(t-2)(t+1) = 0$
$t=2, t=-1$

$\boxed{t=2}$

E
______ 8. What is $\frac{dy}{dx}$ for $r = 6\cos 4\theta$ ?

(A) $-\frac{\cos 4\theta \cos\theta - \sin 4\theta \sin\theta}{\cos 4\theta \sin\theta + \sin 4\theta \cos\theta}$ (B) $\frac{\cos 4\theta \cos\theta - 4\sin 4\theta \sin\theta}{\cos 4\theta \sin\theta + 4\sin 4\theta \cos\theta}$ (C) $-\frac{\cos 4\theta \cos\theta}{\cos 4\theta \sin\theta + 4\sin 4\theta \cos\theta}$

(D) $-\frac{\cos 4\theta \cos\theta - 4\sin 4\theta \sin\theta}{\cos 4\theta \sin\theta}$ (E) $-\frac{\cos 4\theta \cos\theta - 4\sin 4\theta \sin\theta}{\cos 4\theta \sin\theta + 4\sin 4\theta \cos\theta}$

$y = 6\cos 4\theta \sin\theta$, $y' = -24\sin 4\theta \sin\theta + 6\cos 4\theta \cos\theta$
$x = 6\cos 4\theta \cos\theta$, $x' = -24\sin 4\theta \cos\theta - 6\cos 4\theta \sin\theta$

$$\frac{\cancel{6}(\cos 4\theta \cos\theta - 4\sin 4\theta \sin\theta)}{-\cancel{6}(\cos 4\theta \sin\theta + 4\sin 4\theta \cos\theta)}$$

B
______ 9. If $x(t) = \cos(2t)$ and $y(t) = \sin(2t)$, which of the following is equal to $\frac{d^2y}{dx^2}$?

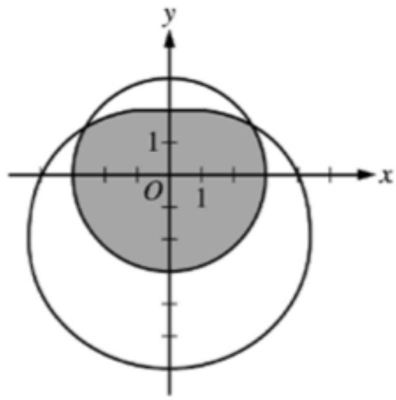
(A) $2\csc^2(2t)$ (B) $-\csc^3(2t)$ (C) $\csc^3(2t)$

(D) $-2\csc^3(2t)$ (E) $-2\csc^2(2t)$

$y' = 2\cos(2t)$
$x' = -2\sin(2t)$

$\frac{dy}{dx} = -\cot(2t)$

$$\frac{d^2y}{dx^2} = \frac{2\csc^2(2t)}{-2\sin(2t)}$$
$$= -\frac{\csc^2(2t)}{\sin(2t)} = -\csc^3(2t)$$




(2013, BC-2)

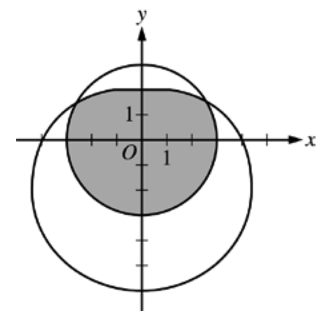
14. The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(a) Let $S$ be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of $S$.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time $t$ seconds, $\theta = t^2$. Find the time $t$ in the interval $1 \le t \le 2$ for which the $x$-coordinate of the particle's position is $-1$.

(c) For the particle described in part (b), find the position vector in terms of $t$. Find the velocity vector at time $t = 1.5$.

### Question 2




The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(a) Let $S$ be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of $S$.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time $t$ seconds, $\theta = t^2$. Find the time $t$ in the interval $1 \le t \le 2$ for which the $x$-coordinate of the particle's position is $-1$.

(c) For the particle described in part (b), find the position vector in terms of $t$. Find the velocity vector at time $t = 1.5$.

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(a) Area $= 6\pi + \frac{1}{2}\int_{\pi/6}^{5\pi/6} (4 - 2\sin\theta)^2 \, d\theta = 24.709$ (or 24.708)

oR

$$2\left[\frac{1}{2}\int_{-\frac{\pi}{2}}^{\pi/6} (3^2)\, d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2} (4 - 2\sin\theta)^2 \, d\theta\right]$$

3 : { 1 : integrand; 1 : limits and constant; 1 : answer }

(b) $x = r\cos\theta \Rightarrow x(\theta) = (4 - 2\sin\theta)\cos\theta$

$x(t) = \left(4 - 2\sin\left(t^2\right)\right)\cos\left(t^2\right)$

$x(t) = -1$ when $t = 1.428$ (or 1.427)

3 : { 1 : $x(\theta)$ or $x(t)$; 1 : $x(\theta) = -1$ or $x(t) = -1$; 1 : answer }

(c) $y = r\sin\theta \Rightarrow y(\theta) = (4 - 2\sin\theta)\sin\theta$

$y(t) = \left(4 - 2\sin\left(t^2\right)\right)\sin\left(t^2\right)$

Position vector $= \langle x(t), y(t) \rangle$

$= \left\langle \left(4 - 2\sin\left(t^2\right)\right)\cos\left(t^2\right), \left(4 - 2\sin\left(t^2\right)\right)\sin\left(t^2\right) \right\rangle$

$v(1.5) = \langle x'(1.5), y'(1.5) \rangle$

$= \langle -8.072, -1.673 \rangle$ (or $\langle -8.072, -1.672 \rangle$)

3 : { 2 : position vector; 1 : velocity vector }

1 A
2 D
3 C

4 B
5 C
6 D

7 C
8 E
9 B