Name KEY Date Period

BC Calculus: TEST 8.1 – 8.6. NO CALCULATOR, NO CALCULATOR

Part I: Multiple Choice

D 1. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) 2/3 (B) 8/3 (C) 4 (D) 14/3 (E) 16/3

B 2. The region in the first quadrant between the *x*-axis and the graph of $y = 6x - x^2$ is rotated around the *y*-axis. The volume of the resulting solid of revolution is given by

(A) $\int_0^6 \pi\left(6x - x^2\right)^2 dx$ (B) $\int_0^6 2\pi x\left(6x - x^2\right) dx$ (C) $\int_0^6 \pi x\left(6x - x^2\right)^2 dx$

(D) $\int_0^6 \pi\left(3 + \sqrt{9 - y}\right)^2 dy$ (E) $\int_0^9 \pi\left(3 + \sqrt{9 - y}\right)^2 dy$

A 3. The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the *x*-axis are equilateral triangles, then its volume is

(A) $\dfrac{\sqrt{3}\left(1 - e^{-6}\right)}{8}$ (B) $\dfrac{\sqrt{3}}{8}e^{-6}$ (C) $\dfrac{\sqrt{3}}{4}e^{-6}$ (D) $\dfrac{\sqrt{3}}{4}e^{-3}$ (E) $\dfrac{\sqrt{3}}{4}\left(1 - e^{-3}\right)$

C 4. What is the length of the arc of $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 3$?

(A) 8/3 (B) 4 (C) 14/3 (D) 16/3 (E) 7

D 5. $\lim\limits_{x \to 0} \dfrac{e^{2x} - 1}{\tan x} =$ (A) -1 (B) 0 (C) 1 (D) 2 (E) DNE

C 6. $\lim\limits_{h \to 0} \dfrac{\int_1^{1+h} \sqrt{x^5 + 8}\,dx}{h} =$ (A) 0 (B) 1 (C) 3 (D) $2\sqrt{2}$ (E) DNE

C 7. $\lim\limits_{x \to \infty} \left(1 + 5e^x\right)^{1/x} =$ (A) 0 (B) 1 (C) e (D) e^5 (E) DNE

A 8. $\int_2^\infty \dfrac{dx}{x^2} =$ (A) $\dfrac{1}{2}$ (B) $\ln 2$ (C) 1 (D) 2 (E) DNE

E 9. $\int_0^1 \dfrac{x + 1}{x^2 + 2x - 3}\,dx =$ (A) $-\ln\sqrt{3}$ (B) $-\dfrac{\ln\sqrt{3}}{2}$ (C) $\dfrac{1 - \ln\sqrt{3}}{2}$ (D) $\ln\sqrt{3}$ (E) Diverges

1 D
2 B
3 A
4 C
5 D
6 C
7 C
8 A
9 E

II. Free Response: Show all work below the line.

10. Let f be the function given by $f(x) = kx^2 - x^3$, where k is a positive constant. Let R be the region in the first quadrant bounded by the graph of f and the x-axis.

(a) Find all values of the constant k for which the area of R equals 2.

(b) For $k > 0$, write, but do not evaluate, an integral expression in terms of k for the volume of the solid generated when R is rotated about the x-axis.

(c) For $k > 0$, write, but do not evaluate, and expression in terms of k, involving one or more integrals that gives the perimeter of R.

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1 D
2 B
3 A
4 C
5 D
6 C
7 C
8 A
9 E

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Question 4

Let f be the function given by $f(x) = kx^2 - x^3$, where k is a positive constant. Let R be the region in the first quadrant bounded by the graph of f and the x-axis.

(a) Find all values of the constant k for which the area of R equals 2.

(b) For $k > 0$, write, but do not evaluate, an integral expression in terms of k for the volume of the solid generated when R is rotated about the x-axis.

(c) For $k > 0$, write, but do not evaluate, an expression in terms of k, involving one or more integrals, that gives the perimeter of R.

(a) For $x \geq 0$, $f(x) = x^2(k - x) \geq 0$ if $0 \leq x \leq k$

$$\int_0^k \left(kx^2 - x^3\right) dx = \left(\frac{k}{3}x^3 - \frac{1}{4}x^4\right)\Bigg|_{x=0}^{x=k} = \frac{k^4}{12}$$

Area $= \dfrac{k^4}{12} = 2;\ k = \sqrt[4]{24}$

$$4 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{value of integral} \\ 1 : \text{answer} \end{cases}$$

(b) Volume $= \pi \displaystyle\int_0^k \left(kx^2 - x^3\right)^2 dx$

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$$

(c) Perimeter $= k + \displaystyle\int_0^k \sqrt{1 + \left(2kx - 3x^2\right)^2}\, dx$

$$3 : \begin{cases} 1 : \int_0^k \sqrt{1 + (f'(x))^2}\, dx \\ 1 : \text{uses } f'(x) = 2kx - 3x^2 \text{ in integrand} \\ 1 : \text{answer} \end{cases}$$