Name____________________________________ Date_________________________ Period________

BC Calculus: Practice TEST: 8.1 through 8.6

Part I: Multiple Choice
NO CALCULATOR ON THIS SECTION

_____ 1. The base of a solid S is the region enclosed by the graph of $y=\sqrt{\ln x}$, the line $x=e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}\left(e^3-1\right)$

_____ 2. When the region enclosed by the graphs the graphs of $y=x$ and $y=4x-x^2$ is revolved about the y-axis, the volume of the solid generated is given by

(A) $\pi\int_0^3\left(x^3-3x^2\right)dx$ (B) $\pi\int_0^3\left(x^2-\left(4x-x^2\right)^2\right)dx$ (C) $\pi\int_0^3\left(3x-x^2\right)^2dx$

(D) $2\pi\int_0^3\left(x^3-3x^2\right)dx$ (E) $2\pi\int_0^3\left(3x^2-x^3\right)dx$

_____ 3. The base of a solid in the region in the first quadrant enclosed by the graph of $y=2-x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

(A) $\pi\int_0^2(2-y)^2\,dy$ (B) $\int_0^2(2-y)\,dy$ (C) $\pi\int_0^{\sqrt{2}}\left(2-x^2\right)^2dx$

(D) $\int_0^{\sqrt{2}}\left(2-x^2\right)^2dx$ (E) $\int_0^{\sqrt{2}}\left(2-x^2\right)dx$

_____ 4. Which of the following integrals gives the length of the graph of $y=\tan x$ between $x=a$ and $x=b$, where $0<a<b<\frac{\pi}{2}$?

(A) $\int_a^b\sqrt{x^2+\tan^2 x}\,dx$ (B) $\int_a^b\sqrt{x+\tan x}\,dx$ (C) $\int_a^b\sqrt{1+\sec^2 x}\,dx$

(D) $\int_a^b\sqrt{1+\tan^2 x}\,dx$ (E) $\int_a^b\sqrt{1+\sec^4 x}\,dx$

_____ 5. A region in the plane is bounded by the graph of $y=\frac{1}{x}$, the x-axis, the line $x=m$, and the line $x=2m$, $m>0$. The area of this region

(A) is independent of m. (B) increases as m increases. (C) decreases as m increases.

(D) decreases as m increases when $m<\frac{1}{2}$; increases as m increases when $m>\frac{1}{2}$.

(E) increases as m increases when $m<\frac{1}{2}$; decreases as m increases when $m>\frac{1}{2}$.

_____ 6. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\frac{8\pi}{3}$

_____ 7. The region R in the first quadrant is encloses by the lines $x = 0$ and $y = 5$ and the graph fo $y = x^2 + 1$. The volume of the solid generated when R is revolved about the y-axis is

(A) 6π (B) 8π (C) $\frac{34\pi}{3}$ (D) 16π (E) $\frac{544\pi}{15}$

_____ 8. The length of the curve $y = x^3$ from $x = 0$ to $x = 2$ is given by

(A) $\int_0^2 \sqrt{1+x^6}\,dx$ (B) $\int_0^2 \sqrt{1+3x^2}\,dx$ (C) $\pi\int_0^2 \sqrt{1+9x^4}\,dx$ (D) $2\pi\int_0^2 \sqrt{1+9x^4}\,dx$ (E) $\int_0^2 \sqrt{1+9x^4}\,dx$

_____ 9. The area of the region enclosed by the graphs of $y = x^2$ and $y = x$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{5}{6}$ (E) 1

_____ 10. $\lim\limits_{x\to 1} \dfrac{\int_1^x e^{t^2}\,dt}{x^2-1} =$ (A) 0 (B) 1 (C) $\frac{e}{2}$ (D) e (E) nonexistent

_____ 11. $\int_0^\infty x^2 e^{-x^3}\,dx =$ (A) $-\frac{1}{3}$ (B) 0 (C) $\frac{1}{3}$ (D) 1 (E) divergent

_____ 12. $\lim\limits_{x\to 1} \dfrac{\ln\left(x^2+4x-4\right)}{5x^2-5} =$ (A) $\frac{8}{5}$ (B) $\frac{6}{5}$ (C) $\frac{3}{5}$ (D) $\frac{7}{10}$ (E) DNE

_____ 13. $\lim\limits_{x\to 0} \dfrac{\sin^{-1}(3x)}{\tan^{-1}(4x)} =$ (A) $\frac{3}{4}$ (B) 0 (C) 4 (D) $\frac{4}{3}$ (E) DNE

_____ 14. $\lim\limits_{x\to 0^+} x(5-6\ln x) =$ (A) 0 (B) -6 (C) -1 (D) $-\infty$ (E) ∞

_____ 15. $\lim\limits_{x\to 5} \left(\dfrac{7}{\ln(x-4)} - \dfrac{7}{x-5}\right) =$ (A) 0 (B) 3.5 (C) 7 (D) $-\infty$ (E) ∞

_____ 16. $\lim\limits_{x\to 0} (1+6x)^{\csc x} =$ (A) e^6 (B) e (C) 6 (D) $-\infty$ (E) ∞

_____ 17. $\displaystyle\int_2^{\infty} \frac{x}{\sqrt[3]{x^2-2}}\,dx =$ (A) $2^{2/3}$ (B) $\dfrac{2^{2/3}}{4}$ (C) $\dfrac{3\cdot 2^{2/3}}{4}$ (D) $-\dfrac{3\cdot 2^{2/3}}{4}$ (E) Diverges

_____ 18. $\displaystyle\int_1^{\infty} \frac{6x}{\left(1+x^2\right)^2}\,dx =$ (A) 1.5 (B) 2 (C) 3 (D) 6 (E) Diverges

_____ 19. $\displaystyle\int_{-\infty}^{\infty} 4xe^{-5x^2}\,dx =$ (A) $\dfrac{4}{5}$ (B) $\dfrac{2}{5}$ (C) $\dfrac{1}{5}$ (D) 0 (E) Diverges

_____ 20. $\displaystyle\int_1^{\infty} \frac{4\arctan x}{1+x^2}\,dx =$ (A) $\dfrac{3}{8}$ (B) $\dfrac{3\pi^2}{8}$ (C) $\dfrac{\pi^2}{2}$ (D) $\dfrac{3\pi^2}{4}$ (E) Diverges

_____ 21. $\displaystyle\int_0^{2} \frac{4}{(x-1)^{1/3}}\,dx =$ (A) 4 (B) 6 (C) 12 (D) 0 (E) Diverges

_____ 22. (Calculator Permitted) Determine the smallest integer a so that $\displaystyle\int_a^{\infty} \frac{2}{x^2+1}\,dx \le \frac{1}{50}$.

(A) 130 (B) 110 (C) 120 (D) 90 (E) 100

Part II. Free Response
(CALCULATOR PERMITTED)

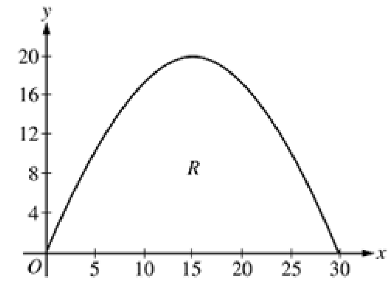
23. (2007-BC1) Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.
(b) Find the volume of the solid generated when R is rotated about the x-axis.
(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this sold.

24. (2003-BC1) Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$.

(a) Find the area of R.
(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.
(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

25. (BC 2009B-1) A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of $y = f(x)$ for $0 \le x \le 30$, where $f(x) = 20\sin\left(\frac{\pi x}{30}\right)$. Both x and y are measured in centimeters. The region R is shown in the figure. The derivative of f is $f'(x) = \frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)$.



(a) The region R is cut out of a 30-centimeter-by-20-centimeter rectangular sheet of cardboard, and the remaining cardboard is discarded. Find the area of the discarded cardboard.

(b) The cake is a solid with base R. Cross sections of the cake perpendicular to the x-axis are semicircles. If the baker uses 0.05 grams of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?

(c) Find the perimeter of the base of the cake.