

AP Calculus: 5.1 – 6.4 Calculator permitted

Part I: Multiple Choice—show all work for credit. No work, no credit. Put the capital letter in the blank to the left of each question number.

_____ 1. A solid is generated when the region in the first quadrant enclosed by the graph of $y = (x^2 + 1)^3$, the line $x = 1$, the x -axis, and the y -axis is revolved about the x -axis. Its volume is found by evaluating which of the following integrals?

- (A) $\pi \int_1^8 (x^2 + 1)^3 dx$ (B) $\pi \int_1^8 (x^2 + 1)^6 dx$ (C) $\pi \int_0^1 (x^2 + 1)^3 dx$ (D) $\pi \int_0^1 (x^2 + 1)^6 dx$ (E) $2\pi \int_0^1 (x^2 + 1)^6 dx$

_____ 2. If $\frac{dy}{dx} = \frac{3x^2 + 2}{y}$, and $y = 4$ when $x = 2$, then when $x = 3$, $y =$

- (A) $\sqrt{66}$ (B) $-\sqrt{66}$ (C) 58 (D) $-\sqrt{58}$ (E) $\sqrt{58}$

_____ 3. The volume generated by revolving about the y -axis the region enclosed by the graphs of $y = 9 - x^2$ and $y = 9 - 3x$, for $0 \leq x \leq 2$, is

- (A) 2π (B) 4π (C) 8π (D) 24π (E) 48π

_____ 4. $\int \ln(2x) dx =$

- (A) $\frac{\ln(2x)}{x} + C$ (B) $\frac{\ln(2x)}{2x} + C$ (C) $x \ln x - x + C$ (D) $x \ln 2x - x + C$ (E) $2x \ln 2x - 2x + C$

_____ 5. Find the distance traveled for $t \in [0, 4]$ seconds for a particle whose velocity, in ft/sec, is given by $v(t) = 7e^{-t^2}$.

- (A) 0.976 (B) 6.204 (C) 6.359 (D) 12.720 (E) 7.000

_____ 6. Find the area of the region bounded by the graphs of the $f(x) = e^{-x^2/4}$ and $y = 0.5$.

- (A) 0.516 (B) 0.480 (C) 0.240 (D) 1.032 (E) 1.349

_____ 7. The base of a solid S is the region enclosed by the graphs of $4x + 5y = 20$, the x -axis, and the y -axis. If the cross-sections of S perpendicular to the x -axis are semicircles, then the volume of S is

- (A) $\frac{5\pi}{3}$ (B) $\frac{10\pi}{3}$ (C) $\frac{50\pi}{3}$ (D) $\frac{225\pi}{3}$ (E) $\frac{425\pi}{3}$

_____ 8. $\int \frac{18x-17}{(2x-3)(x+1)} dx =$

- (A) $8\ln|2x-3| + 7\ln|x+1| + C$ (B) $2\ln|2x-3| + 7\ln|x+1| + C$ (C) $4\ln|2x-3| + 7\ln|x+1| + C$
(D) $7\ln|2x-3| + 2\ln|x+1| + C$ (E) $\frac{7}{2}\ln|2x-3| + 4\ln|x+1| + C$

_____ 9. Use Euler's Method with $\Delta x = 0.2$ to approximate $y(1)$ if $\frac{dy}{dx} = y$ and $y(0) = 1$.

- (A) 1.200 (B) 2.075 (C) 2.488 (D) 5.513 (E) 3.872

_____ 10. Which of the following gives the best approximation of the length of the arc of $y = \cos(2x)$ from

$x = 0$ to $x = \frac{\pi}{4}$?

- (A) 0.785 (B) 0.955 (C) 1.0 (D) 1.318 (E) 1.977

Part I: Free Response—show all work in the space provided for credit. Notation, notation, notation. Clearly communicate your results. Include units on all final numeric and verbal answers.

10.

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t , $0 \leq t \leq 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t , $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6} \int_0^6 C(t) dt$. Using correct units, explain the meaning of $\frac{1}{6} \int_0^6 C(t) dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.