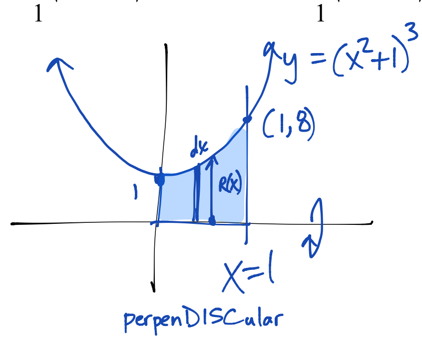
Name KEY Date ______ Deciduous Tree ______

AP Calculus: 5.1 – 6.4 Calculator permitted

Part I: Multiple Choice—show all work for credit. No work, no credit. Put the capital letter in the blank to the left of each question number.

D 1. A solid is generated when the region in the first quadrant enclosed by the graph of $y=\left(x^2+1\right)^3$, the line $x=1$, the x-axis, and the y-axis is revolved about the x-axis. Its volume is found by evaluating which of the following integrals?

(A) $\pi\int_1^8\left(x^2+1\right)^3dx$ (B) $\pi\int_1^8\left(x^2+1\right)^6dx$ (C) $\pi\int_0^1\left(x^2+1\right)^3dx$ (D) $\pi\int_0^1\left(x^2+1\right)^6dx$ (E) $2\pi\int_0^1\left(x^2+1\right)^6dx$



$$\text{Vol}=\pi\int_0^1\left(R(x)\right)^2dx$$
$$=\pi\int_0^1\left((x^2+1)^3-0\right)^2dx$$
$$=\pi\int_0^1\left(x^2+1\right)^6dx$$

E 2. If $\dfrac{dy}{dx}=\dfrac{3x^2+2}{y}$, and $y=4$ when $x=2$, then when $x=3$, $y=$

(A) $\sqrt{66}$ (B) $-\sqrt{66}$ (C) 58 (D) $-\sqrt{58}$ (E) $\sqrt{58}$

$\int y\,dy=\int(3x^2+2)\,dx$

$\frac{1}{2}y^2=x^3+2x+C$

$y^2=2x^3+4x+C$

$y=\pm\sqrt{2x^3+4x+c}$ (GenSoln)

at $(2,+4)$: $4=\pm\sqrt{2(8)+8+c}$

$16=16+8+c$

$c=-8$

So, $y=+\sqrt{2x^3+4x-8}$ (Particular Soln)

since $y=+4$ when $x=2$.

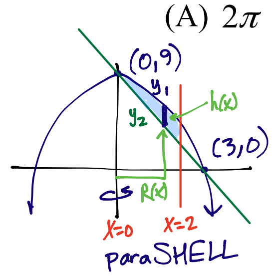
So, when $x=3$: $y=+\sqrt{2(27)+12-8}$

$y=\sqrt{54+4}$

$y=\sqrt{58}$

C 3. The volume generated by revolving about the y-axis the region enclosed by the graphs of $y_1=9-x^2$ and $y_2=9-3x$, for $0\le x\le 2$, is

(A) 2π (B) 4π (C) 8π (D) 24π (E) 48π



$$\text{Vol}=2\pi\int_a^b R(x)\cdot h(x)\cdot dx$$
$$=2\pi\int_0^2(x)\left((9-x^2)-(9-3x)\right)dx$$
$$=8\pi$$

D 4. $\int \ln(2x)\,dx =$

(A) $\dfrac{\ln(2x)}{x}+C$ (B) $\dfrac{\ln(2x)}{2x}+C$ (C) $x\ln x - x + C$ (D) $x\ln 2x - x + C$ (E) $2x\ln 2x - 2x + C$

$u = \ln 2x \qquad dv = dx$

$du = \frac{1}{2x}(2) \qquad v = x$

$\int \ln(2x)\,dx = x\ln(2x) - \int \left(\frac{1}{2x}\right)(2)(x)\,dx$

$x\ln(2x) - \int 1\,dx$

$x\ln(2x) - x + C$

B 5. Find the distance traveled for $t \in [0,4]$ seconds for a particle whose velocity, in ft/sec, is given by $v(t) = 7e^{-t^2}$.

(A) 0.976 (B) 6.204 (C) 6.359 (D) 12.720 (E) 7.000

$\int_a^b v(t)\,dt = \text{Displacement}$

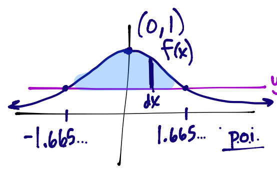
$\int_a^b |v(t)|\,dt = \text{Dist Travelled}$

So, $\text{Dist} = \int_0^4 \left|7e^{-t^2}\right| dt$

$= 6.203$ or 6.204

D 6. Find the area of the region bounded by the graphs of the $f(x) = e^{-x^2/4}$ and $y = 0.5$.

(A) 0.516 (B) 0.480 (C) 0.240 (D) 1.032 (E) 1.349



$e^{-x^2/4} = 0.5$

$x = 1.665\ldots = A$

$\text{Area} = \int_{-A}^{A} (f(x) - 0.5)\,dx$

or

$= 2\int_0^A \left(e^{-x^2/4} - 0.5\right)dx$ (symmetry)

$= 1.032$

B 7. The base of a solid S is the region enclosed by the graphs of $4x + 5y = 20$, the x-axis, and the y-axis. If the cross-sections of S perpendicular to the x-axis are semicircles, then the volume of S is

(A) $\dfrac{5\pi}{3}$ (B) $\dfrac{10\pi}{3}$ (C) $\dfrac{50\pi}{3}$ (D) $\dfrac{225\pi}{3}$ (E) $\dfrac{425\pi}{3}$

$5y = -4x + 20$

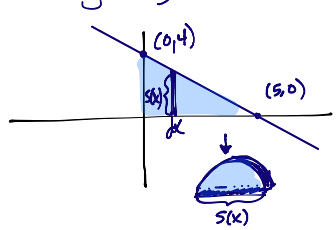
$y = -\frac{4}{5}x + 4$

x-int:

$-\frac{4}{5}x + 4 = 0$

$4 = \frac{4}{5}x$

$x = 5$



$\text{Vol} = \frac{\pi}{8}\int_0^5 (S(x))^2\,dx$

$= \frac{\pi}{8}\int_0^5 \left(\left(-\frac{4}{5}x + 4\right) - 0\right)^2 dx$

$= \frac{10\pi}{3}$

B 8. $\int \frac{18x-17}{(2x-3)(x+1)}dx =$ partial fraction decomposition

(A) $8\ln|2x-3|+7\ln|x+1|+C$ (B) $2\ln|2x-3|+7\ln|x+1|+C$ (C) $4\ln|2x-3|+7\ln|x+1|+C$

(D) $7\ln|2x-3|+2\ln|x+1|+C$ (E) $\frac{7}{2}\ln|2x-3|+4\ln|x+1|+C$

$$\int\left[\frac{\frac{18(\frac{3}{2})-17}{3/2+1}}{2x-3}+\frac{\frac{18(-1)-17}{-2-3}}{x+1}\right]dx$$

$$\int\left[\frac{\frac{10}{5/2}}{2x-3}+\frac{\frac{-35}{-5}}{x+1}\right]dx$$

$$\int\left[\frac{4}{2x-3}+\frac{7}{x+1}\right]dx$$

$(4)(\frac{1}{2})\ln|2x-3|+7\ln|x+1|+c$

$2\ln|2x-3|+7\ln|x+1|+c$

C 9. Use Euler's Method with $\Delta x = 0.2$ to approximate $y(1)$ if $\frac{dy}{dx}=y$ and $y(0)=1$.

(A) 1.200 (B) 2.075 (C) 2.488 (D) 5.513 (E) 3.872

$\Delta x = 0.2$

x	y	$\frac{dy}{dx}=y=m$	$\Delta y = m\Delta x$	$y_{new}=y+\Delta y$
0	1	1	0.2	1+0.2=1.2
0.2	1.2	1.2	0.24	1.2+0.24=1.44
0.4	1.44	1.44	0.288	1.44+0.288=1.728
0.6	1.728	1.728	0.3456	1.728+0.3456=2.0736
0.8	2.0736	2.0736	0.41472	2.0736+0.41472=2.48832
1	2.48832			

So, $y(1)\approx 2.48832$

D 10. Which of the following gives the best approximation of the length of the arc of $y=\cos(2x)$ from $x=0$ to $x=\frac{\pi}{4}$?

(A) 0.785 (B) 0.955 (C) 1.0 (D) 1.318 (E) 1.977

$y' = -2\sin(2x)$

$$\mathcal{L} = \int_0^{\pi/4}\sqrt{1+(-2\sin(2x))^2}\,dx$$

$= 1.317591791$

≈ 1.318

Part I: Free Response—show all work in the space provided for credit. Notation, notation, notation. Clearly communicate your results. Include units on all final numeric and verbal answers.

10.

$\Delta x = 2$ $\Delta x = 2$ $\Delta x = 2$

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

$C'(3.5) \approx \dfrac{12.8 - 11.2}{4 - 3}$ ounces/minute ($\checkmark_2$)

or 1.6 oz/min

($\checkmark_1$) in presence of difference quotient

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

To prove existence of a slope/Deriv/m → MVT

$C'(t) = \dfrac{C(4) - C(2)}{4-2}$ ($\checkmark_3$)

$C'(t) = \dfrac{12.8 - 8.8}{4-2}$

$C'(t) = \frac{4}{2}$

So, $C'(t) = 2$ for some $t \in (2,4)$ by MVT

($\checkmark_4$)

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

$\frac{1}{6}\int_0^6 C(t)\,dt \approx \left(\frac{1}{6}\right)(2)[5.3 + 11.2 + 13.8]$

$= \frac{1}{3}(30.3)$ ($\checkmark_5$) method

$= 10.1$ ($\checkmark_6$) numeric answer

$\frac{1}{6}\int_0^6 C(t)\,dt$ gives the average number of ounces in the cup from $t = 0$ minutes to $t = 6$ minutes.

($\checkmark_7$) interpretation with units (oz & min!!)

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

($\checkmark_9$)

($\checkmark_8$) Notation $B'(5) = 0.866$ oz/min OR $= \dfrac{6.4}{e^2}$

(Calculator: $\boxed{\text{Math}}$ $\boxed{8}$)