Name KEY Date________________ Period________

AP Calculus: TEST: 7.1 – 8.2 Calculator permitted

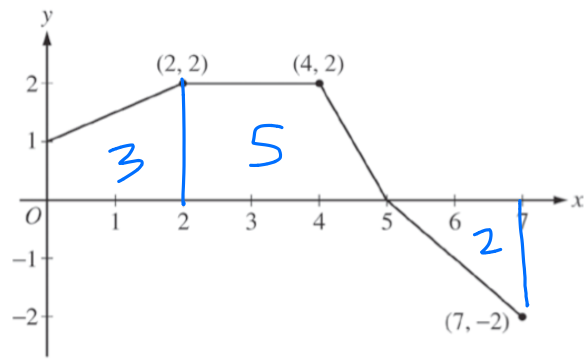
Part I: Multiple Choice

A 1. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 - 4$ for time $t \geq 0$. If the particle is at position $x = -2$ at time $t = 0$, what is the position of the particle at time $t = 3$?

(A) 13 (B) 15 (C) 16 (D) 17 (E) 25

A 2. The graph of a function f is shown at right. What is the value of $\int_0^7 f(x)\,dx$?

(A) 6 (B) 8 (C) 10 (D) 14 (E) 18



Graph of f

C 3. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

$\int e^{-y} = \int e^x$
$-e^{-y} = e^x + c$
$e^{-y} = -e^x + c$
$-y = \ln(c - e^x)$
$y = -\ln(c - e^x)$
$-\ln 4 = -\ln(c - 1$
$4 = c - 1$
$c = 5$
$y = -\ln(5 - e^x)$

(A) $y = -x - \ln 4$ (B) $y = x - \ln 4$ (C) $y = -\ln(-e^x + 5)$

(D) $y = -\ln(e^x + 3)$ (E) $y = \ln(e^x + 3)$

E 4. For time $t \geq 0$, the position of a particle traveling along a line is given by a differentiable function s. If s is increasing for $0 \leq t < 2$ and s is decreasing for $t > 2$, which of the following is the total distance the particle travels for $0 \leq t \leq 5$?

(A) $s(0) + \int_0^2 s'(t)\,dt - \int_2^5 s'(t)\,dt$ (B) $s(0) + \int_2^5 s'(t)\,dt - \int_0^2 s'(t)\,dt$ (C) $\int_2^5 s'(t)\,dt - \int_0^2 s'(t)\,dt$

(D) $\left|\int_0^5 s'(t)\,dt\right|$ (E) $\int_0^5 |s'(t)|\,dt$

A 5. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit ($^\circ F$). If the initial temperature of the tea, at time $t = 0$ minutes, is $200^\circ F$ and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

(A) $175^\circ F$ (B) $130^\circ F$ (C) $95^\circ F$ (D) $70^\circ F$ (E) $45^\circ F$

B 6. The rate at which customers arrive at a counter to be served is given by $F(t) = 12 + 6\cos\left(\frac{t}{\pi}\right)$ for $0 \le t \le 60$, where $F(t)$ is measured in customers per minute and t is measured in minutes. To the nearest whole number, how many customers arrive at the counter over the 60-minute period?

(A) 720 (B) 725 (C) 732 (D) 744 (E) 756

A 7. The population P of a city grows according to the differential equation $\frac{dP}{dt} = kP$, where k is a constant and t is measured in years. If the population of the city doubles every 12 years, what is the value of k?

$2 = e^{k12}$

(A) 0.058 (B) 0.061 (C) 0.167 (D) 0.693 (E) 8.318

$\frac{\ln 2}{12} = k$

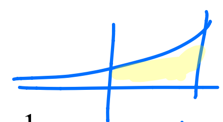
t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

C 8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

$50 + 3(6.2) + 5(5.9) + 3(5.6) =$

(A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

A 9. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

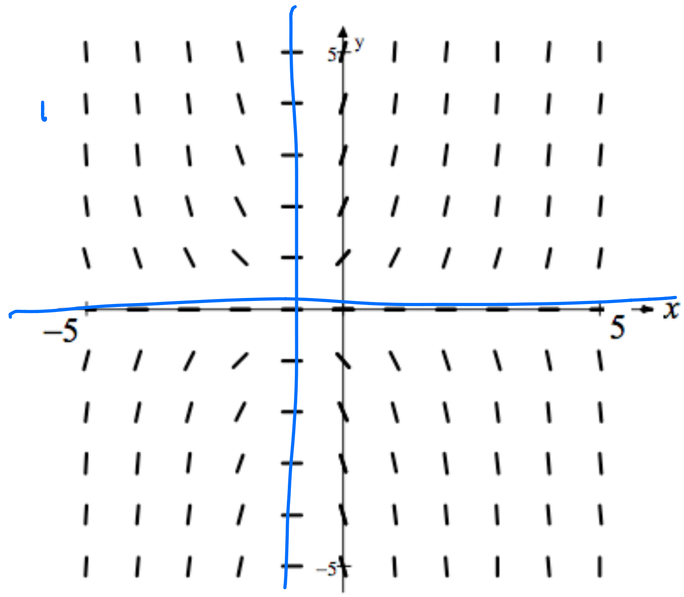


(A) $2e - 2$ (B) $2e$ (C) $\frac{e}{2} - 1$ (D) $\frac{e-1}{2}$ (E) $e - 1$

$x = 2$

$\rightarrow 3.436$

C
_____ 10.



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = xy$ (B) $\frac{dy}{dx} = xy - y$ (C) $\frac{dy}{dx} = xy + y$ $y(x+1)$ (D) $\frac{dy}{dx} = xy + x$ (E) $\frac{dy}{dx} = (x+1)^3$

Part II: **Free Response**: Show all work below the problem in the space provided. Round to 3 decimals when applicable and include units when applicable, and wear galoshes when applicable.

AB 2012 #1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

11. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is $55^\circ F$. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computation that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

AP® CALCULUS AB
2012 SCORING GUIDELINES

Question 1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

(a) $W'(12) \approx \dfrac{W(15) - W(9)}{15 - 9} = \dfrac{67.9 - 61.8}{6}$ $\dfrac{67.9-61.8}{15-9}$

$= 1.017$ (or 1.016)

The water temperature is increasing at a rate of approximately 1.017 °F per minute at time $t = 12$ minutes.

$2: \begin{cases} 1: \text{estimate} \\ 1: \text{interpretation with units} \end{cases}$

1 A
2 A
3 C
4 E

5 A
6 B
7 A
8 C
9 A

10 C

(b) $\int_0^{20} W'(t)\,dt = W(20) - W(0) = 71.0 - 55.0 = 16$

The water has warmed by 16 °F over the interval from $t = 0$ to $t = 20$ minutes.

$2: \begin{cases} 1: \text{value} \\ 1: \text{interpretation with units} \end{cases}$

(c) $\dfrac{1}{20}\int_0^{20} W(t)\,dt \approx \dfrac{1}{20}(4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15))$

$= \dfrac{1}{20}(4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9)$ ✓

$= \dfrac{1}{20} \cdot 1215.8 = 60.79$

This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.

$3: \begin{cases} 1: \text{left Riemann sum} \\ 1: \text{approximation} \\ 1: \text{underestimate with reason} \end{cases}$

(d) $W(25) = 71.0 + \int_{20}^{25} W'(t)\,dt$

$= 71.0 + 2.043155 = 73.043$

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$