Name______________________________ Date_________________ Alchemy Secret________

BC Calculus: TEST 5.1 – 6.1, NO CALCULATOR

Part I: Multiple Choice—Show all work on scratch paper and attach to the back.

E 1. If $G(x)$ is an antiderivative for $f(x)$ and $G(2)=-7$, then $G(4)=$

(A) $f'(4)$ (B) $-7+f'(4)$ (C) $\int_2^4 f(t)dt$ (D) $\int_2^4 (-7+f(t))dt$ (E) $-7+\int_2^4 f(t)dt$

$G'=f$

$\int f\,dx = G$

$G(4)=G(2)+\int_2^4 f(x)\,dx$

$= -7+\int_2^4 f(x)\,dx$

B 2. $\int x\sin(6x)dx=$

(A) $-x\cos(6x)+\sin(6x)+C$ (B) $-\frac{x}{6}\cos(6x)+\frac{1}{36}\sin(6x)+C$ (C) $-\frac{x}{6}\cos(6x)+\frac{1}{6}\sin(6x)+C$

(D) $\frac{x}{6}\cos(6x)+\frac{1}{36}\sin(6x)+C$ (E) $6x\cos(6x)-\sin(6x)+C$

$u=x$ $dv=\sin(6x)$

$du=dx$ $v=-\frac{1}{6}\cos(6x)$

$-\frac{x}{6}\cos(6x)-(-\frac{1}{6})\int\cos(6x)\,dx$

$-\frac{x}{6}\cos(6x)+\frac{1}{36}\sin(6x)+c$

D 3. Given that $y(1)=-3$ and $\frac{dy}{dx}=2x+y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of 0.5, starting at $x=1$?

$\Delta x=0.5$

(A) -5 (B) -4.25 (C) -4 (D) -3.75 (E) -3.5

x	y	m	$m\Delta x = \Delta y$	ynew
1	-3	-1	-0.5	-3.5
1.5	-3.5	-0.5	-.25	-3.75
2	-3.75			

so $y(2)\approx -3.75$

B 4. If $\int x^2\cos x\,dx = f(x)-\int 2x\sin x\,dx$, then $f(x)=$

(A) $2\sin x+2x\cos x+C$ (B) $x^2\sin x+C$ (C) $2x\cos x-x^2\sin x+C$

(D) $4\cos x-2x\sin x+C$ (E) $(2-x^2)\cos x-4\sin x+C$

$u=x^2$ $dv=\cos x\,dx$

$du=2x\,dx$ $v=\sin x$

$\int x^2\cos x\,dx = x^2\sin x-\int 2x\sin x\,dx$

D 5. If the graph of $y=f(x)$ contains the point $(0,2)$, $\frac{dy}{dx}=\frac{-x}{ye^{x^2}}$ and $f(x)>0$ for all x, then

$f(x)=$

(A) $3+e^{-x^2}$ (B) $\sqrt{3}+e^{-x}$ (C) $1+e^{-x}$ (D) $\sqrt{3+e^{-x^2}}$ (E) $\sqrt{3+e^{x^2}}$

$\int y\,dy=\int -xe^{-x^2}dx$

$\frac{1}{2}y^2=\frac{1}{2}e^{-x^2}+c$

$y^2=e^{-x^2}+c$

$y=\pm\sqrt{e^{-x^2}+c}$

at (0,2):

$2=\pm\sqrt{e^0+c}$

$4=1+c$

$c=3$

so $y=+\sqrt{e^{-x^2}+3}$

since $(0,+2)$

A 6. Population y grows according to the equation $\frac{dy}{dt}=ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) $\ln\sqrt[10]{2}$ (B) $\frac{1}{5}$ (C) $\ln\sqrt{10}$ (D) $2\ln 10$ (E) 5

$y=Ce^{kt}$

$(0,1)$
$(10,2)$ create arbitrary initial value & another pt based on rule of growth

$y=e^{kt}$, $c=1$

@ (10,2): $2=e^{10k}$

$\ln 2=10k$

$k=\frac{1}{10}\ln 2=\ln\sqrt[10]{2}$

$k=0.069314$

B 7. $\int\frac{x^2}{3+4x+x^2}dx=\int\frac{x^2}{x^2+4x+3}dx$

(A) $1+\frac{9}{2}\ln|x+3|-\frac{1}{2}\ln|x+1|+C$ (B) $x-\frac{9}{2}\ln|x+3|+\frac{1}{2}\ln|x+1|+C$ (C) $x+\frac{9}{2}\ln|x+3|-\frac{1}{2}\ln|x+1|+C$

(D) $x-\frac{9}{2}\ln|x+3|-\frac{1}{2}\ln|x+1|+C$ (E) $1+\frac{9}{2}\ln|x+3|+\frac{1}{2}\ln|x+1|+C$

$\int\left[1+\frac{-4x-3}{x^2+4x+3}\right]dx$

$\int\left[1+\frac{-4x-3}{(x+1)(x+3)}\right]dx$

$\int\left[1+\frac{1/2}{x+1}+\frac{9/-2}{x+3}\right]dx$

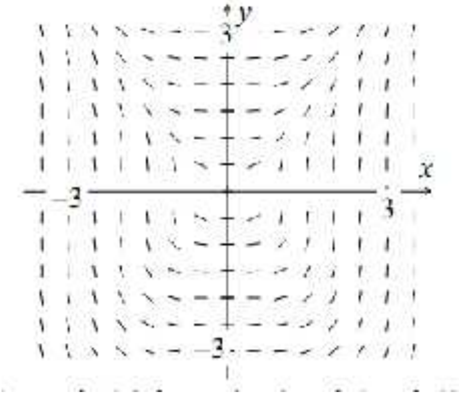
$x+\frac{1}{2}\ln|x+1|-\frac{9}{2}\ln|x+3|+c$

$$x^2+4x+3\,\overset{1}{\overline{)\,x^2}}$$

$-x^2\mp 4x\mp 3$

$-4x-3$

E 8. Show below is a slope field for which of the following differential equations?



zero slopes
when $x=0$
(y-axis)

pos slopes when $x>0$
QI, QIV

neg slopes when $x<0$
QII, QIII

(x must have odd exponent

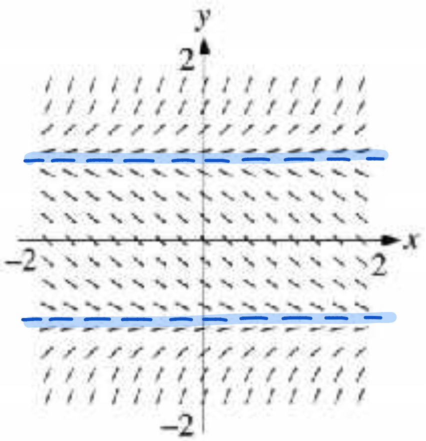
(y must have even exponent)

(A) $\frac{dy}{dx}=\frac{x}{y}$ (B) $\frac{dy}{dx}=\frac{x^2}{y^2}$ (C) $\frac{dy}{dx}=\frac{x^3}{y}$ (D) $\frac{dy}{dx}=\frac{x^2}{y}$ (E) $\frac{dy}{dx}=\frac{x^3}{y^2}$

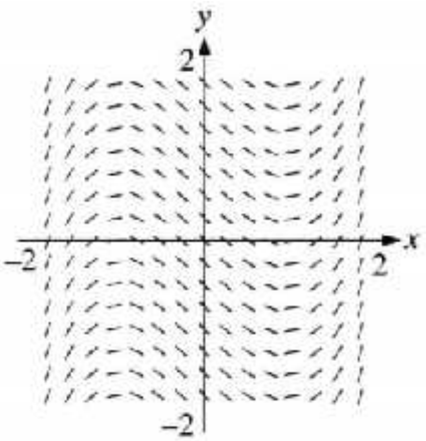
A 9. Which of the following could be the slope field for the differential equation $\frac{dy}{dx}=y^2-1$?

$y^2-1=0$
$y=\pm 1$ (zero slopes along horz lines $y=\pm 1$

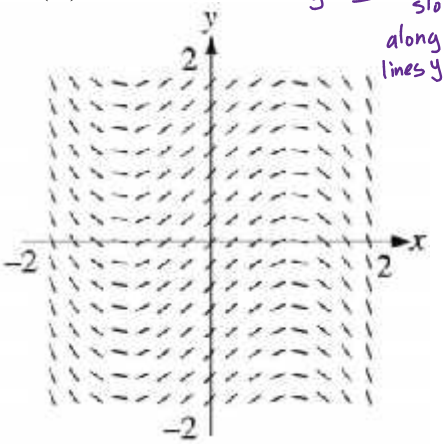
(A)



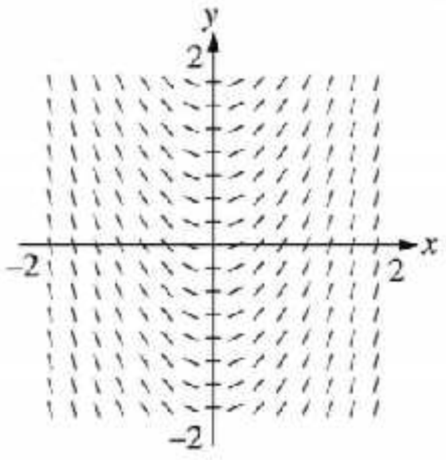
(B)



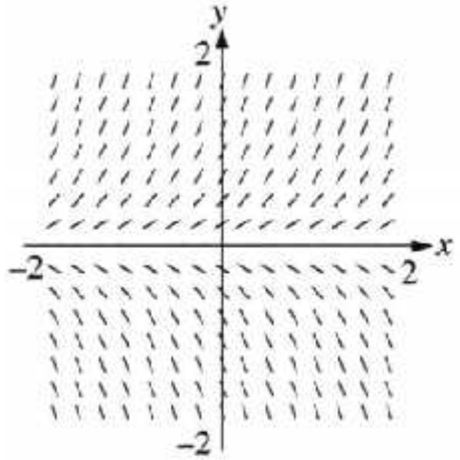
(C)



(D)



(E)



E 10. $\frac{d}{dx}\left(\int_0^{x^2}\sin\left(t^3\right)dt\right)=\sin\left(x^2\right)^3\cdot 2x-\sin\left(0^3\right)\cdot 0=2x\sin\left(x^6\right)$

(A) $-\cos\left(x^6\right)$ (B) $\sin\left(x^3\right)$ (C) $\sin\left(x^6\right)$ (D) $2x\sin\left(x^3\right)$ (E) $2x\sin\left(x^6\right)$

2nd FTOC

Part II: Free Response—Show all work in the space provided

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

11. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t I smeasured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to estimate $C'(3.5)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

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Question 3

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

(a) $C'(3.5) \approx \dfrac{C(4) - C(3)}{4 - 3} = \dfrac{12.8 - 11.2}{1} = 1.6$ ounces/min

$$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{units} \end{cases}$$

(b) C is differentiable $\Rightarrow$ C is continuous (on the closed interval)

$$\frac{C(4) - C(2)}{4 - 2} = \frac{12.8 - 8.8}{2} = 2$$

Therefore, by the Mean Value Theorem, there is at least one time t, $2 < t < 4$, for which $C'(t) = 2$.

$$2 : \begin{cases} 1 : \dfrac{C(4) - C(2)}{4 - 2} \\ 1 : \text{conclusion, using MVT} \end{cases}$$

(c)
$$\begin{aligned} \frac{1}{6}\int_0^6 C(t)\,dt &\approx \frac{1}{6}[2 \cdot C(1) + 2 \cdot C(3) + 2 \cdot C(5)] \\ &= \frac{1}{6}(2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8) \\ &= \frac{1}{6}(60.6) = 10.1 \text{ ounces} \end{aligned}$$

$\frac{1}{6}\int_0^6 C(t)\,dt$ is the average amount of coffee in the cup, in ounces, over the time interval $0 \le t \le 6$ minutes.

$$3 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{approximation} \\ 1 : \text{interpretation} \end{cases}$$

(d) $B'(t) = -16(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$

$B'(5) = 6.4e^{-0.4(5)} = \dfrac{6.4}{e^2}$ ounces/min

$$2 : \begin{cases} 1 : B'(t) \\ 1 : B'(5) \end{cases}$$