Name_____ Date_____ Period_____

Calculus TEST: 6.4 to 8.1. NO Calculator permitted

Part I: **Multiple Choice**: Put the correct CAPITAL letter (yes I was shouting) in the blank to the left of the question number.

_____ 1. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$ (B) $e^{\tan x} + 5$ (C) $5e^{\tan x}$ (D) $\tan x + 5$ (E) $\tan x + 5e^x$

_____ 2. Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

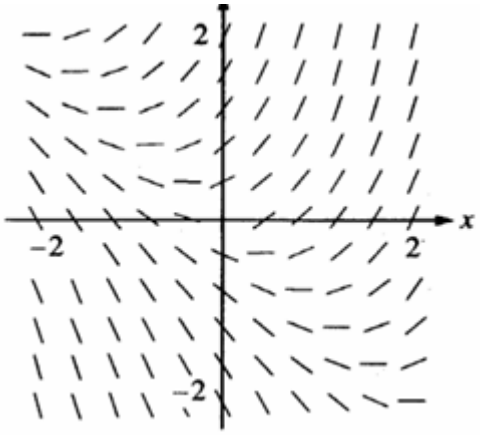
(A) $\frac{3\ln 3}{\ln 2}$ (B) $\frac{2\ln 3}{\ln 2}$ (C) $\frac{\ln 3}{\ln 2}$ (D) $\ln\left(\frac{27}{2}\right)$ (E) $\ln\left(\frac{9}{2}\right)$

_____ 3. If $\frac{dy}{dt} = ky$ and *k* is a nonzero constant, then *y* could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$

(D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

_____4. Shown at right is a slope field for which of the following differential equations?



(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$

(D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

_____ 5. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4 - x^2}} =$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{1}{2}\ln 2$ (E) $-\ln 2$

_____ 6. $\int_0^8 \frac{dx}{\sqrt{1 + x}} =$ (A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

_____ 7. A kangaroo moves in a straigt line so that its velocity at time $t \geq 0$ on a horizontal line is $t - t^2$. What is the *total* distance covered by the kangaroo between $t = 0$ and $t = 2$?

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) 5

_____ 8. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at $t = 1$?

(A) 4 (B) 6 (C) 9 (D) 11 (E) 12

_____ 9. The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table at right. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

(A) 26 ft/sec (B) 30 ft/sec (C) 37 ft/sec (D) 39 ft/sec (E) 41 ft/sec

_____ 10. Let $F(x)$ be an antiderivative of $\sin x \cos^2 x$. If $F\left(\frac{\pi}{2}\right) = 0$, then $F(0) =$

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1

_____ 11. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0,2]$?

(A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$ (d) $\frac{52}{3}$ (E) 24

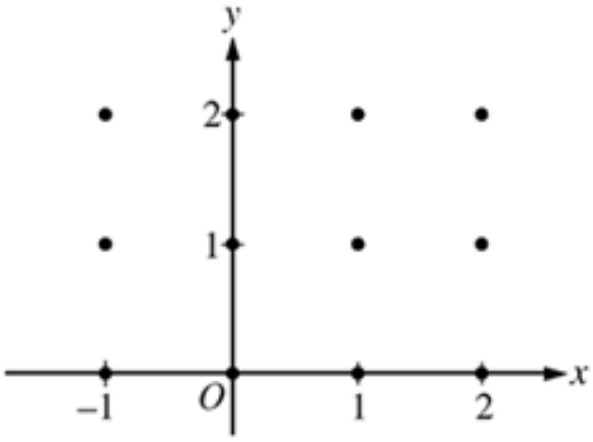
_____ 12. $\int_{-2}^{2} \left(x^7 + k\right) dx = 16$, then $k =$

(A) -12 (B) -4 (C) 0 (D) 4 (E) 12

Part II: **Free Response**: Show all work below the problem in the space provided. Round to 3 decimals when applicable and include units when applicable, and wear galoshes when applicable.

13. (2005-BC4) Consider the differential equation $\frac{dy}{dx} = 2x - y$

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and sketch the solution curve that passes through the point $(0,1)$.



(b) The solution curve that passes through the point $(0,1)$ has a local minimum at $x = \ln\left(\frac{3}{2}\right)$. What is the y-coordinate of this local minimum?

(c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use a tangent line approximation centered at $x = 0$ to approximate $f(-0.4)$. Show the work that leads to your answer.

(d) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.