Name____ Date______________ Angle of Depression_______

AP Calculus TEST 4.1-5.1, No Calculator

Section 1: Multiple Choice—You know what to do

_____1. $\int \left(x^2-2\right)^2 dx =$

(A) $\left(\dfrac{x^3}{3}-2x\right)^2+C$

(B) $\dfrac{\left(x^2-2\right)^3}{6x}+C$

(C) $\dfrac{x^5}{5}-\dfrac{4x^3}{3}+4x+C$

(D) $\dfrac{2x}{3}\left(x^2-2\right)^3+C$

(E) $\left(\dfrac{x^2-2}{3}\right)^3+C$

_____ 2. At each point (x, y) on a curve, $\dfrac{d^2y}{dx^2}=6x$. Additionally, the line $y=6x+4$ is tangent to the curve at $x=-2$. Which of the following is an equation of the curve that satisfies these conditions?

(A) $y=6x^2-32$

(B) $y=x^3-6x-12$

(C) $y=2x^3-3x$

(D) $y=x^3-6x+12$

(E) $y=2x^3+3x-12$

_____ 3. $\int \dfrac{\sin 2x}{\cos x} dx =$

(A) $-2\cos x+C$ (B) $2\cos x+C$ (C) $-\cos 2x+C$ (D) $\cos x+C$ (E) $\cos 2x+C$

_____ 4. $\int \tan^2 x dx =$

(A) $\tan x+x+C$ (B) $\sec x+x+C$ (C) $\sec x-x+C$ (D) $\tan x-x+C$ (E) $\tan x+C$

_____ 5. $\int \frac{2x^2}{\sqrt{x^3+3}}dx =$

(A) $\frac{2}{3}\sqrt{x^3+3}+C$ (B) $\frac{4}{3\sqrt{x^2+3}}+C$ (C) $\frac{4}{3}\sqrt{x^3+3}+C$ (D) $\frac{1}{3}\sqrt{x^3+3}+C$ (E) $\frac{3}{4}\sqrt{x^3+3}+C$

_____ 6. $\int x\sqrt{x-1}dx =$

(A) $\frac{2}{5}(x-1)^{5/2}+\frac{2}{3}(x-1)^{3/2}+C$

(B) $\frac{1}{2}(x-1)^4+C$

(C) $\frac{5}{2}(x-1)^{5/2}+\frac{3}{2}(x-1)^{3/2}+C$

(D) $\frac{1}{3}x^2(x-1)^{3/2}+C$

(E) $\frac{2}{3}\left(x^2-x\right)^{3/2}+C$

_____ 7. $\int \tan^3 x\cdot\sec^2 x dx =$

(A) $\frac{1}{2}\tan^2 x+C$ (B) $\frac{1}{4}\tan^4 x+C$ (C) $\frac{1}{2}\sec^2 x+C$ (D) $\frac{\sec^3 x\cdot\tan^4 x}{12}+C$ (E) $4\tan^4 x+C$

_____ 8. What is the average value of $\cos x$ on the closed interval $\left[0,\frac{\pi}{2}\right]$?

(A) $\frac{1}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{1}{2\pi}$ (D) $\frac{2}{\pi}$ (E) $\frac{\pi}{2}$

_____ 9. $\frac{d}{dx}\left[\int_{2x}^{x^2}\cos^2 t dt\right]=$

(A) $2x\left[\cos^2\left(x^2\right)-\cos^2(2x)\right]$

(B) $\cos^2\left(x^2\right)$

(C) $2x^2\cos^2\left(x^2\right)$

(D) $2\left[x\cos^2\left(x^2\right)-\cos^2(2x)\right]$

(E) $\cos^2\left(x^2\right)-\cos^2(2x)$

Part II: Free Response—Do and show all work in the space provided. Have fun!

10.

At time $t=0$, a boiled potato is take from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius $(°C)$ at time $t=0$, and the internal temperature of the potato is greater than $27°C$ for all times $t>0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt}=-\frac{1}{4}(H-27)$, where $H(t)$ is measured in degrees Celsius and $H(0)=91$.

(a) Write an equation for the line tangent to the graph of H at $t=0$. Use this equation to approximate the internal temperature of the potato at time $t=3$.

(b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t=3$.

(c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?