Name KEY Date________________ Angle of Depression________

AP Calculus TEST 4.1-5.1, No Calculator

Section 1: Multiple Choice—You know what to do

C 1. $\int (x^2-2)^2\,dx =$

(A) $\left(\frac{x^3}{3}-2x\right)^2+C$

(B) $\frac{(x^2-2)^3}{6x}+C$

(C) $\frac{x^5}{5}-\frac{4x^3}{3}+4x+C$

(D) $\frac{2x}{3}(x^2-2)^3+C$

(E) $\left(\frac{x^2-2}{3}\right)^3+C$

$\int (x^2-2)^2 dx$

$\int (x^4-4x^2+4)\,dx$

$\frac{1}{5}x^5-\frac{4}{3}x^3+4x+C$

B 2. At each point (x,y) on a curve, $\frac{d^2y}{dx^2}=6x$. Additionally, the line $y=6x+4$ is tangent to the curve at $x=-2$. Which of the following is an equation of the curve that satisfies these conditions?

(A) $y=6x^2-32$

(B) $y=x^3-6x-12$

(C) $y=2x^3-3x$

(D) $y=x^3-6x+12$

(E) $y=2x^3+3x-12$

$\frac{d^2y}{dx^2}=6x$

$\frac{dy}{dx}=3x^2+C$

for $y'(-2)=6$: $6=3(-2)^2+C$

$6=12+C$

$C=-6$

So, $\frac{dy}{dx}=3x^2-6$

$\frac{dy}{dx}=3x^2-6$

$y=x^3-6x+C$

for $y(-2)=-8$: $-8=(-2)^3-6(-2)+C$

$-8=-8+12+C$

$C=-12$

So, $y=x^3-6x-12$

@ $x=-2$: $y=6x+4$

$y'=6$

So, $y'(-2)=6$

$y(-2)=6(-2)+4$

$=-12+4$

$=-8$

So, $y(-2)=-8$

A 3. $\int \frac{\sin 2x}{\cos x}\,dx =$

(A) $-2\cos x+C$ (B) $2\cos x+C$ (C) $-\cos 2x+C$ (D) $\cos x+C$ (E) $\cos 2x+C$

$\int \frac{\sin 2x}{\cos x}dx$

$\int \frac{2\sin x\cos x}{\cos x}dx$

$2\int \sin x\,dx=-2\cos x+C$

D 4. $\int \tan^2 x\,dx =$

(A) $\tan x+x+C$ (B) $\sec x+x+C$ (C) $\sec x-x+C$ (D) $\tan x-x+C$ (E) $\tan x+C$

$\int \tan^2 x\,dx$

$\int (\sec^2 x-1)\,dx$

$\tan x-x+C$

$2\int x^2(x^3+3)^{-1/2}\,dx$

$3x^2$

C 5. $\displaystyle\int \frac{2x^2}{\sqrt{x^3+3}}\,dx =$ $(2)\left(\frac{1}{3}\right)(2)(x^3+3)^{1/2}+C$

$\frac{4}{3}\sqrt{x^3+3}+C$

(A) $\frac{2}{3}\sqrt{x^3+3}+C$ (B) $\frac{4}{3\sqrt{x^2+3}}+C$ (C) $\frac{4}{3}\sqrt{x^3+3}+C$ (D) $\frac{1}{3}\sqrt{x^3+3}+C$ (E) $\frac{3}{4}\sqrt{x^3+3}+C$

A 6. $\int x\sqrt{x-1}\,dx =$

(A) $\frac{2}{5}(x-1)^{5/2}+\frac{2}{3}(x-1)^{3/2}+C$

(B) $\frac{1}{2}(x-1)^4+C$

(C) $\frac{5}{2}(x-1)^{5/2}+\frac{3}{2}(x-1)^{3/2}+C$

(D) $\frac{1}{3}x^2(x-1)^{3/2}+C$

(E) $\frac{2}{3}\left(x^2-x\right)^{3/2}+C$

$u=x-1$

$x=u+1$

$dx=du$

$\int(u+1)u^{1/2}\,du$

$\int\left(u^{3/2}+u^{1/2}\right)du$

$\frac{2}{5}u^{5/2}+\frac{2}{3}u^{3/2}+C$

$\frac{2}{5}(x-1)^{5/2}+\frac{2}{3}(x-1)^{3/2}+C$

B 7. $\int \tan^3 x\cdot\sec^2 x\,dx =$

(A) $\frac{1}{2}\tan^2 x+C$ (B) $\frac{1}{4}\tan^4 x+C$ (C) $\frac{1}{2}\sec^2 x+C$ (D) $\frac{\sec^3 x\cdot\tan^4 x}{12}+C$ (E) $4\tan^4 x+C$

$\int(\tan x)^3\sec^2 x\,dx$

$\frac{1}{4}\tan^4 x+C$

D 8. What is the average value of $\cos x$ on the closed interval $\left[0,\frac{\pi}{2}\right]$?

(A) $\frac{1}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{1}{2\pi}$ (D) $\frac{2}{\pi}$ (E) $\frac{\pi}{2}$

Avg value $= \dfrac{\int_0^{\pi/2}\cos x\,dx}{\pi/2-0} = \frac{2}{\pi}\left(\sin x\Big|_0^{\pi/2}\right) = \frac{2}{\pi}\left[\sin\frac{\pi}{2}-\sin 0\right]$

$= \frac{2}{\pi}[1-0]$

$= \frac{2}{\pi}$

D 9. $\displaystyle\frac{d}{dx}\left[\int_{2x}^{x^2}\cos^2 t\,dt\right] =$ $\cos^2(x^2)\cdot 2x-\cos^2(2x)\cdot 2$

$2\left[x\cos^2(x^2)-\cos^2(2x)\right]$

(A) $2x\left[\cos^2\left(x^2\right)-\cos^2(2x)\right]$

(B) $\cos^2\left(x^2\right)$

(C) $2x^2\cos^2\left(x^2\right)$

(D) $2\left[x\cos^2\left(x^2\right)-\cos^2(2x)\right]$

(E) $\cos^2\left(x^2\right)-\cos^2(2x)$

Part II: Free Response—Do and show all work in the space provided. Have fun!

10.

At time $t = 0$, a boiled potato is take from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius $\left({}^\circ C\right)$ at time $t = 0$, and the internal temperature of the potato is greater than $27^\circ C$ for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

(a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

pt: $(0,91)$

$$\text{Slope} = \left.\frac{dH}{dt}\right|_{(0,91)} = -\frac{1}{4}(91-27) = -\frac{1}{4}(64) = -16 \quad \checkmark_1$$

$$\text{So } y = \underbrace{\mathcal{L}(x) = 91 - 16(x-0)}_{\checkmark_2}$$

$$\text{So, } H(3) \approx \mathcal{L}(3) = 91 - 16(3) = 91 - 48 = \underbrace{43^\circ C}_{\checkmark_3}$$

(b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

$$\frac{dH}{dt} = -\frac{1}{4}(H-27)$$

$$\frac{d}{dt}: \frac{d^2H}{dt^2} = -\frac{1}{4}\left(\frac{dH}{dt}\right) = -\frac{1}{4}\left(-\frac{1}{4}(H-27)\right) = \frac{1}{16}(H-27)$$

$$\left.\frac{d^2H}{dt^2}\right|_{(0,91)} = \frac{1}{16}(91-27) = \frac{64}{16} = 4 > 0$$

++ $\checkmark_4$ Answer with reason

So, $\mathcal{L}(3)$ underapproximates $H(3)$

(c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G-27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

$$\frac{dG}{dt} = -(G-27)^{2/3}$$

$$(G-27)^{-2/3}\,dG = -dt \quad \sqrt{5}$$

$$\int (G-27)^{-2/3}\,dG = \int (-1)\,dt$$

$$3(G-27)^{1/3} = -t + C \quad \sqrt{6}$$

$$(G-27)^{1/3} = -\tfrac{1}{3}t + C$$

$$G - 27 = \left(-\tfrac{1}{3}t + C\right)^3$$

$$G(t) = \left(-\tfrac{1}{3}t + C\right)^3 + 27$$

$\sqrt{7}$ for $G(0) = 91$: $91 = (0 + C)^3 + 27$

$$64 = C^3$$

$$C = \sqrt[3]{64}$$

$$C = 4$$

$$\text{So, } G(t) = \left(-\tfrac{1}{3}t + 4\right)^3 + 27 \quad \sqrt{8}$$

$$G(3) = \left(-\tfrac{1}{3}(3) + 4\right)^3 + 27\ ^\circ C$$

$$= (-1 + 4)^3 + 27\ ^\circ C$$

$$= 3^3 + 27\ ^\circ C$$

$$= 27 + 27\ ^\circ C$$

$$= 54\ ^\circ C \quad \sqrt{9}$$