AP Calculus TEST 4.1-5.1, No Calculator

Section 1: Multiple Choice—You know what to do

1.
$$\int (x^2 - 2)^2 dx = \int (x^2 - 2)^2 dx$$

$$\left(-2x\right)^{2}+C$$

$$\int (x^{2} - 2)^{2} dx = \int (x^{2} - 2)^{2} dx$$
(A) $\left(\frac{x^{3}}{3} - 2x\right)^{2} + C$

$$\int (x^{2} - 4x^{2} + 4) dx$$

$$\int (x^{2} - 4x^{2} + 4) dx$$

$$\int \int (x^{2} - 4x^{2} + 4) dx$$

$$\int \int \int (x^{2} - 2)^{2} dx = \int (x^{2} - 4x^{2} + 4) dx$$

(B)
$$\frac{\left(x^2 - 2\right)^3}{6x} + C$$

(C)
$$\frac{x^5}{5} - \frac{4x^3}{3} + 4x + C$$

(D)
$$\frac{2x}{3}(x^2-2)^3 + C$$

$$(E)\left(\frac{x^2-2}{3}\right)^3+C$$

2. At each point (x, y) on a curve, $\frac{d^2y}{dx^2} = 6x$. Additionally, the line y = 6x + 4 is tangent to the curve at

x = -2. Which of the following is an equation of the curve that satisfies these conditions?

(A)
$$y = 6x^2 - 32$$

(B)
$$y = x^3 - 6x - 12$$

(B)
$$y = x^3 - 6x - 1$$

(D)
$$v = r^3 - 6r + 13$$

(D)
$$y = x^3 - 6x + 12$$

(E)
$$y = 2x^3 + 3x - 12$$

$$\frac{dy}{dy} = 3x^2 + C$$

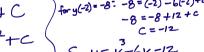
$$= 3x + C$$

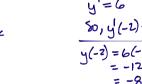
$$= 3x + C$$

$$-8 = -8 + 12$$

$$-8 = -8 + 12$$

$$C = -12$$





$$x = -2. \text{ Which of the following is an equation of the curve that satisfies these conditions?}$$

$$(A) \ y = 6x^2 - 32$$

$$(B) \ y = x^3 - 6x - 12$$

$$(C) \ y = 2x^3 - 3x$$

$$(D) \ y = x^3 - 6x + 12$$

$$(E) \ y = 2x^3 + 3x - 12$$

$$\int \frac{\sin 2x}{\cos x} dx = \int \frac{\cos 2x}{\cos x} dx = \int \frac$$

(A) $-2\cos x + C$ (B) $2\cos x + C$ (C) $-\cos 2x + C$ (D) $\cos x + C$ (E) $\cos 2x + C$

$$4. \int \tan^2 x dx =$$

 $\int \frac{\sin 2x}{\cos x} dx =$

(B)
$$\sec x + x + C$$

(B)
$$\sec x + x + C$$
 (C) $\sec x - x + C$ (D) $\tan x - x + C$ (E) $\tan x + C$

$$2\int_{X^{2}(x^{3}+3)}^{y_{2}} dx$$

$$5. \int_{\sqrt{x^{3}+3}}^{2x^{2}} dx = (2)(\frac{1}{3})(2)(x^{3}+3) + C$$

$$\int \frac{1}{\sqrt{x^3 + 3}} dx = \frac{(2)(\frac{1}{3})(2)(x+3)}{\frac{1}{3}\sqrt{x^3 + 3}} + C$$
(A) $\frac{2}{3}\sqrt{x^3 + 3} + C$ (B) $\frac{4}{3\sqrt{x^2 + 3}} + C$ (C) $\frac{4}{3}\sqrt{x^3 + 3} + C$ (D) $\frac{1}{3}\sqrt{x^3 + 3} + C$ (E) $\frac{3}{4}\sqrt{x^3 + 3} + C$

$$\triangle$$
 6. $\int x\sqrt{x-1}dx =$

(A)
$$\frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$$
 $\chi = \omega + 1$

(B)
$$\frac{1}{2}(x-1)^4 + C$$

(C)
$$\frac{5}{2}(x-1)^{5/2} + \frac{3}{2}(x-1)^{3/2} + C$$
 $\int_{\zeta} (u^{\frac{3}{2}} + u^{\frac{1}{2}}) du$
(D) $\frac{1}{3}x^{2}(x-1)^{3/2} + C$ $\int_{\zeta} (u^{\frac{3}{2}} + u^{\frac{1}{2}}) du$
 $\frac{2}{5}(x-1)^{\frac{3}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + C$

(D)
$$\frac{1}{3}x^2(x-1)^{3/2} + C$$

(E)
$$\frac{2}{3}(x^2-x)^{3/2}+C$$

$$\int \int \tan^3 x \cdot \sec^2 x dx =$$

(A)
$$\frac{1}{2} \tan^2 x + C$$
 (B) $\frac{1}{4} \tan^4 x + C$ (C) $\frac{1}{2} \sec^2 x + C$ (D) $\frac{\sec^3 x \cdot \tan^4 x}{12} + C$ (E) $4 \tan^4 x + C$
 $\int \frac{\tan^3 x}{4} + C$ (E) $4 \tan^4 x + C$

$$8$$
. What is the average value of $\cos x$ on the closed interval $\left[0, \frac{\pi}{2}\right]$?

$$(A) \frac{1}{2} \qquad (B) \frac{\pi}{4} \qquad (C) \frac{1}{2\pi} \qquad (D) \frac{2}{\pi} \qquad (E) \frac{\pi}{2}$$

$$A \vee g \vee a |_{R} = \underbrace{\int_{D}^{T/2} L_0 S \times d \times}_{T/2 - D} = \underbrace{\frac{2}{\pi} \left(S \ln \times \right)_{0}^{T/2}}_{T/2 - D} = \underbrace{\frac{2}{\pi} \left[1 - 6 \right]}_{=\frac{2}{\pi}}$$

$$9. \frac{d}{dx} \left[\int_{2x}^{x^2} \cos^2 t dt \right] = \cos^2(x^2) \cdot 2x - \cos^2(2x) \cdot 2$$

$$\frac{dx \begin{bmatrix} 2x \\ 2x \end{bmatrix}}{(A) 2x \left[\cos^2(x^2) - \cos^2(2x)\right]} \qquad Z \left[\chi \cos^2(x^2) - \cos^2(2x) \right]$$

(B)
$$\cos^2(x^2)$$

(C)
$$2x^2 \cos^2(x^2)$$

(D)
$$2\left[x\cos^2(x^2) - \cos^2(2x)\right]$$

(E)
$$\cos^2\left(x^2\right) - \cos^2\left(2x\right)$$

10.

At time t = 0, a boiled potato is take from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time t = 0, and the internal temperature of the potato is greater than $27^{\circ}C$ for all times t > 0. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where H(t) is measured in degrees Celsius and H(0) = 91.

(a) Write an equation for the line tangent to the graph of H at t = 0. Use this equation to approximate the internal temperature of the potato at time t = 3.

pt:
$$(0,91)$$

 $5lope = \frac{dH}{dt}\Big|_{(0,91)} = -\frac{1}{4}(91-27)$
 $= -\frac{1}{4}(64)$
 $= -16\sqrt{1}$
 $50 y = 2(x) = 91 - 16(x - 0)$

So, $H(3) \approx \mathcal{L}(3) = 91 - 16(3)$ = 91 - 48= 43° C

(b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time t = 3.

(c) For t < 10, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G-27)^{2/3}$, where G(t) is measured in degrees Celsius and G(0) = 91. Find an expression for G(t). Based on this model, what is the internal temperature of the potato at time t = 3?

$$\frac{dG}{dt} = -(G-27)^{\frac{7}{3}}$$

$$(G-27)^{\frac{7}{3}}dG = -dt \sqrt{6}$$

$$\int (G-27)^{\frac{7}{3}}dG = \int (-1)^{\frac{7}{3}}dG$$

$$\int (G-27)^{\frac{7}{3}} = -t + C$$

$$(G-27)^{\frac{7}{3}} = -\frac{1}{3}t + C$$

$$G-27 = (-\frac{1}{3}t + c)^{\frac{7}{3}} + 27$$

$$G(t) = (-\frac{1}{3}t + c)^{\frac{7}{3}} + 27$$

t time
$$t = 3$$
?

$$\frac{dG}{dt} = -(G - 27)^{\frac{7}{3}}$$

$$\frac{dG}{dt} = -(G - 27)^{\frac{7}{3}}$$

$$\frac{dG}{dt} = -(G - 27)^{\frac{7}{3}}$$

$$\frac{dG}{dt} = -dt \sqrt{5}$$

$$\frac{G}{(G - 27)^{\frac{7}{3}}}$$

$$\frac{G}{(G$$