Name____ Date________________ Angle of Incidence_________

AP Calculus TEST 4.1-5.1, No Calculator

Section 1: Multiple Choice—You know what to do

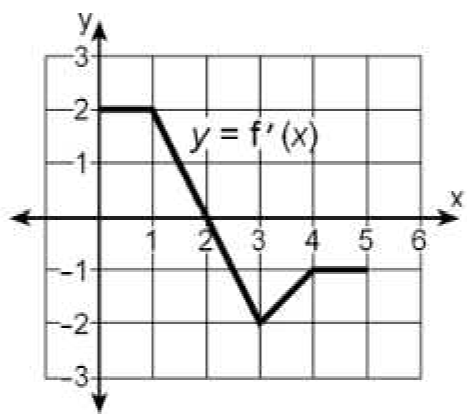
_____1. What is the average value of $f(x)=\sqrt{x}$ on the closed interval $[0,3]$?

(A) $\frac{\sqrt{3}}{3}$ (B) $\frac{3\sqrt{3}}{2}$ (C) $\frac{2\sqrt{3}}{3}$ (D) $2\sqrt{3}$ (E) $\sqrt{3}$

_____ 2. If $F(x)=\int_0^x \sqrt{t^2-9}\,dt$, then $F'(5)=$

(A) $\frac{5}{4}$ (B) -4 (C) $-\frac{5}{4}$ (D) 4 (E) 16

_____ 3. The graph below sows the graph of f', the derivative of function f.



If $f(5)=2$, then $f(0)=$

(A) $\frac{3}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{5}{2}$ (D) 2 (E) $\frac{1}{2}$

_____ 4. If the function f is integrable on $[-a,a]$ and $f(-x)=f(x)$, then which of the following must be true?

(A) $\int_{-a}^{a} f(x)\,dx = \frac{1}{2}\int_{0}^{a} f(x)\,dx$ (B) $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ (C) $\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} f(x)\,dx$

(D) $\int_{-a}^{0} f(x)\,dx = -\int_{0}^{a} f(x)\,dx$ (E) $\int_{-a}^{a} f(x)\,dx = \frac{1}{2}\int_{0}^{a} f(x)\,dx$

_____ 5. $\int x\sqrt{x-1}\;dx =$

(A) $\frac{2}{5}(x-1)^{5/2}+\frac{2}{3}(x-1)^{3/2}+C$ (B) $\frac{1}{2}(x-1)^{4}+C$ (C) $\frac{5}{2}(x-1)^{5/2}+\frac{3}{2}(x-1)^{3/2}+C$

(D) $\frac{1}{3}x^{2}(x-1)^{3/2}+C$ (E) $\frac{2}{3}(x^{2}-x)^{3/2}+C$

_____ 6. $\frac{d}{dx}\left[\int_{2x}^{x^2} \cos^2 t\;dt\right] =$

(A) $2x\left[\cos^2(x^2)-\cos^2(2x)\right]$ (B) $2\left[x\cos^2(x^2)-\cos^2(2x)\right]$ (C) $2x^2\cos^2(x^2)$

(D) $\cos^2(x^2)-\cos^2(2x)$ (E) $\cos^2(x^2)$

_____ 7. $\int \left(x^2+2x+1\right)^{10} dx =$

(A) $\dfrac{(x+1)^{19}}{19}+C$ (B) $\dfrac{1}{11}\left(\dfrac{x^3}{3}+x^2+x\right)^{11}+C$ (C) $\dfrac{\left(x^2+2x+1\right)^{11}}{11}+C$

(D) $\dfrac{(x+1)^{21}}{21}+C$ (E) $\dfrac{(x+1)^{13}}{13}+C$

_____ 8. $\int \left(x^2+1\right)\sqrt[5]{\left(x^3+3x-7\right)^3}\,dx =$

(A) $\dfrac{5}{16}\left(x^2+1\right)\left(x^3+3x-7\right)^{8/5}+C$ (B) $\dfrac{5}{24}\left(x^3+3x-7\right)^{8/5}+C$ (C) $\dfrac{1}{3}\left(x^3+3x-7\right)^{-2/5}+C$

(D) $\dfrac{5}{8}\left(x^3+3x-7\right)^{8/5}+C$ (E) $\dfrac{8}{15}\left(x^3+3x-7\right)^{8/5}+C$

_____ 9. If $\int_{9}^{19} f(x)\,dx = a$, then $\int_{5}^{10} f(2x-1)\,dx =$

(A) $10a$ (B) $\dfrac{a}{2}$ (C) $10(2a-1)$ (D) $2a-1$ (E) $2a$

_____ 10. Let function g be defined by $g(x)=\begin{cases}\sin 2x, & x\le 0\\ \cos\dfrac{x}{2}, & x>0\end{cases}$. Evaluate $\int_{-\text{®}}^{\text{®}} g(x)\,dx$.

(A) 2 (B) 4 (C) 0 (D) -2 (E) -1

Part II: Free Response—Do and show all work in the space provided. Have fun!

11. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y. For $x \geq 1$ and $y \geq 1$, is $\frac{d^2y}{dx^2}$ positive or negative?

(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the tangent line to the graph of $y = f(x)$ at $x = 2$.

(c) Use your equation from part (a) to approximate $f(2.1)$. Is your approximation an over- or an under-approximation of $f(2.1)$. Justify.

(d) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.