

Name _____ Date _____ Shadow Puppet _____

AP Calculus TEST: 4.1—4.4 **No Calculator**

PART I: Multiple Choice. SHOW ALL WORK AND/OR INTEGRAL SET-UPS. NO WORK, NO CREDIT. Put the Capital Letter of the correct answer choice in the space to the left of each problem number.

____ 1. $\int x\sqrt{5x^2 - 4} dx =$

- (A) $\frac{1}{10}(5x^2 - 4)^{3/2} + C$ (B) $\frac{1}{15}(5x^2 - 4)^{3/2} + C$ (C) $-\frac{1}{5}(5x^2 - 4)^{1/2} + C$
(D) $\frac{20}{3}(5x^2 - 4)^{3/2} + C$ (E) $\frac{3}{20}(5x^2 - 4)^{3/2} + C$

____ 2. The average value of the function $f(x) = (x - 1)^2$ on the interval from $x = 1$ to $x = 5$ is

- (A) $\frac{5}{3}$ (B) $\frac{16}{3}$ (C) $\frac{64}{3}$ (D) $\frac{66}{3}$ (E) $\frac{256}{3}$

____ 3. $\int \frac{dx}{9+x^2} =$

- (A) $3\tan^{-1}\left(\frac{x}{3}\right) + C$ (B) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$ (C) $\frac{1}{9}\tan^{-1}\left(\frac{x}{3}\right) + C$ (D) $\frac{1}{3}\tan^{-1}(x) + C$ (E) $\frac{1}{9}\tan^{-1}(x) + C$

____ 4. $\int x\sqrt{x+3} dx =$

(A) $\frac{2}{3}x^{3/2} + 6x^{1/2} + C$ (B) $\frac{2(x+3)^{3/2}}{3} + C$ (C) $\frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$
(D) $\frac{2}{3}(x+3)^{3/2} + 6(x+3)^{1/2} + C$ (E) $\frac{3(x+3)^{3/2}}{2} + C$

____ 5. $\int \frac{3}{x^2 + 4x + 8} dx =$

(A) $\frac{3}{2} \arctan\left(\frac{x+2}{2}\right) + C$ (B) $3 \arcsin\left(\frac{x+2}{2}\right) + C$ (C) $3 \ln|x^2 + 4x + 8| + C$
(D) $\frac{3}{x+2} + C$ (E) $\frac{3}{2}(x^2 + 4x + 8)^2 + C$

____ 6. $\int \frac{2x^2 + x + 18}{x^2 + 9} dx =$

(A) $\frac{1}{4} \ln|2x^2 + x + 18| + C$ (B) $\frac{1}{3} \arctan(x^2 + 9) + C$ (C) $\frac{1}{8}(2x^2 + x + 18)^2 + C$
(D) $2x - \frac{1}{2} \ln|x^2 + 9| + C$ (E) $2x + \frac{1}{2} \ln|x^2 + 9| + C$

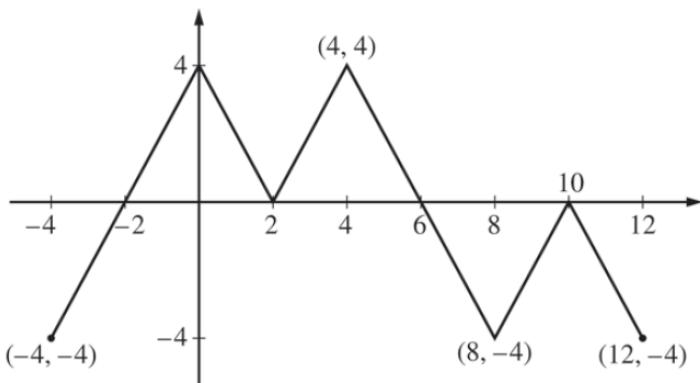
____ 7. If $F(x) = \int_{\pi}^{x^2} \sqrt{1+t^3} dt$, then $F'(x) =$
(A) $\sqrt{1+x^3}$ (B) $2x\sqrt{1+x^3}$ (C) $\sqrt{1+x^6}$ (D) $2x\sqrt{1+x^6}$ (E) $2x\sqrt{1+x^5}$

____ 8. $\int \tan^6 x \cdot \sec^2 x \, dx =$
(A) $\frac{\tan^7 x}{7} + C$ (B) $\frac{\tan^7 x}{7} + \frac{\sec^3 x}{3} + C$ (C) $\frac{\tan^7 x \cdot \sec^3 x}{21} + C$ (D) $\tan^7 x + C$ (E) $\frac{2}{7} \tan^7 x \cdot \sec x + C$

____ 9. $\int_0^1 \tan x \, dx =$
(A) 0 (B) $\frac{\tan^2 1}{2}$ (C) $\ln(\cos(1))$ (D) $\ln(\sec(1))$ (E) $\ln(\sec(1)) - 1$

PART II: Free Response. SHOW ALL WORK AND/OR INTEGRAL SET-UPS in the space provided.
Focus on notation, notation, notation. Communicate, clearly, your result.

10.



Graph of f

The figure above shows the graph of the piecewise-linear function f . For $-4 \leq x \leq 12$, the function

$$F(x) = \int_2^x f(t) dt.$$

(a) Evaluate $F(2)$, $F(8)$, and $F(-4)$. Show the work that leads to your answer.

(b) Find $F'(x)$ and $F'(0)$.

(c) Find the x -coordinate of any local minimum of $F(x)$. Justify.

(d) Does the graph of $F(x)$ have an inflection point at $x = 4$? Justify.

