Name KEY Date___________ Shadow Puppet_____

AP Calculus TEST: 4.1—4.4 **No Calculator**

PART I: Multiple Choice. SHOW ALL WORK AND/OR INTEGRAL SET-UPS. NO WORK, NO CREDIT. Put the Capital Letter of the correct answer choice in the space to the left of each problem number.

B 1. $\int x\sqrt{5x^2-4}\,dx =$

(A) $\frac{1}{10}\left(5x^2-4\right)^{3/2}+C$ (B) $\frac{1}{15}\left(5x^2-4\right)^{3/2}+C$ (C) $-\frac{1}{5}\left(5x^2-4\right)^{1/2}+C$

(D) $\frac{20}{3}\left(5x^2-4\right)^{3/2}+C$ (E) $\frac{3}{20}\left(5x^2-4\right)^{3/2}+C$

$\int x(5x^2-4)^{1/2}\,dx$

$10x$

$\left(\frac{1}{10}\right)\left(\frac{2}{3}\right)(5x^2-4)^{3/2}+C$

$\frac{1}{15}(5x^2-4)^{3/2}+C$

B 2. The average value of the function $f(x)=(x-1)^2$ on the interval from $x=1$ to $x=5$ is

(A) $\frac{5}{3}$ (B) $\frac{16}{3}$ (C) $\frac{64}{3}$ (D) $\frac{66}{3}$ (E) $\frac{256}{3}$

Avg value $= \dfrac{\int_1^5 (x-1)^2\,dx}{5-1}$ or

$= \frac{1}{4}\int_1^5 (x-1)^2\,dx$

$= \left(\frac{1}{4}\right)\left(\frac{1}{3}\right)(x-1)^3\Big|_1^5$

$= \frac{1}{12}\left[(5-1)^3-(1-1)^3\right]$

$= \frac{1}{12}\left[4^3-0^3\right]$

$= \frac{64}{12} = \frac{16}{3}$

$= \frac{1}{4}\int_1^5 (x^2-2x+1)\,dx$

$= \frac{1}{4}\left[\frac{1}{3}x^3-x^2+x\Big|_1^5\right]$

$= \frac{1}{4}\left[\left(\frac{125}{3}-25+5\right)-\left(\frac{1}{3}-1+1\right)\right]$

$= \frac{16}{3}$

B 3. $\int \frac{dx}{9+x^2} =$

(A) $3\tan^{-1}\left(\frac{x}{3}\right)+C$ (B) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$ (C) $\frac{1}{9}\tan^{-1}\left(\frac{x}{3}\right)+C$ (D) $\frac{1}{3}\tan^{-1}(x)+C$ (E) $\frac{1}{9}\tan^{-1}(x)+C$

$\int \frac{1}{x^2+9}\,dx$

$u=x$ $a=3$

$u'=1$

$= \frac{1}{3}\arctan\left(\frac{x}{3}\right)+C$

$= \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$

C 4. $\int x\sqrt{x+3}\,dx =$

Linear, off by $x \rightarrow$ u-sub

(A) $\frac{2}{3}x^{3/2}+6x^{1/2}+C$ (B) $\frac{2(x+3)^{3/2}}{3}+C$ (C) $\frac{2}{5}(x+3)^{5/2}-2(x+3)^{3/2}+C$

(D) $\frac{2}{3}(x+3)^{3/2}+6(x+3)^{1/2}+C$ (E) $\frac{3(x+3)^{3/2}}{2}+C$

Let $u=x+3$
$du=dx$
$x=u-3$

so, $\int x(x+3)^{1/2}\,dx$

$\int (u-3)(u^{1/2})\,du$

$\int (u^{3/2}-3u^{1/2})\,du$

$\frac{2}{5}u^{5/2}-(3)(\frac{2}{3})u^{3/2}+c$

$\frac{2}{5}(x+3)^{5/2}-2(x+3)^{3/2}+c$

A 5. $\int \frac{3}{x^2+4x+8}\,dx =$

(A) $\frac{3}{2}\arctan\left(\frac{x+2}{2}\right)+C$ (B) $3\arcsin\left(\frac{x+2}{2}\right)+C$ (C) $3\ln\left|x^2+4x+8\right|+C$

(D) $\frac{3}{x+2}+C$ (E) $\frac{3}{2}\left(x^2+4x+8\right)^2+C$

$3\int \frac{1}{x^2+4x+8}\,dx$

$3\int \frac{1}{x^2+4x+\underline{4}+\underline{-4}+8}\,dx$ (complete the square)

½ then square

$3\int \frac{1}{(x+2)^2+4}\,dx$

$3\int \frac{1}{(x+2)^2+4}\,dx$

$u=x+2 \quad a=2$
$u'=1$

$(3)(\frac{1}{2})\arctan\left(\frac{x+2}{2}\right)+c$

$\frac{3}{2}\arctan\left(\frac{x+2}{2}\right)+c$

E 6. $\int \frac{2x^2+x+18}{x^2+9}\,dx =$ degree of numerator $\geq$ degree of denominator $\rightarrow$ Long Division

(A) $\frac{1}{4}\ln\left|2x^2+x+18\right|+C$ (B) $\frac{1}{3}\arctan\left(x^2+9\right)+C$ (C) $\frac{1}{8}\left(2x^2+x+18\right)^2+C$

(D) $2x-\frac{1}{2}\ln\left|x^2+9\right|+C$ (E) $2x+\frac{1}{2}\ln\left|x^2+9\right|+C$

$$\begin{array}{r|l} & 2 \\ x^2+9 & 2x^2+x+18 \\ & -2x^2 \quad\quad -18 \\ \hline & x \end{array}$$

so, $\int \frac{2x^2+x+18}{x^2+9}\,dx$

$\int \left(2+\frac{x}{x^2+9}\right)dx$

$\int \left(2+x(x^2+9)^{-1}\right)dx$

$2x$

$2x+(\frac{1}{2})\ln|x^2+9|+c$

D ____ 7. If $F(x)=\int_{\pi}^{x^2}\sqrt{1+t^3}\,dt$, then $F'(x)=$ FTOC part II

(A) $\sqrt{1+x^3}$ (B) $2x\sqrt{1+x^3}$ (C) $\sqrt{1+x^6}$ (D) $2x\sqrt{1+x^6}$ (E) $2x\sqrt{1+x^5}$

$F'(x)=\sqrt{1+(x^2)^3}\,(2x)-\sqrt{1+(\pi)^3}\cdot 0$

$2x\sqrt{1+x^6}-0$

$2x\sqrt{1+x^6}$

A ____ 8. $\int \tan^6 x\cdot\sec^2 x\,dx=$

(A) $\frac{\tan^7 x}{7}+C$ (B) $\frac{\tan^7 x}{7}+\frac{\sec^3 x}{3}+C$ (C) $\frac{\tan^7 x\cdot\sec^3 x}{21}+C$ (D) $\tan^7 x+C$ (E) $\frac{2}{7}\tan^7 x\cdot\sec x+C$

$\int (\tan x)^6\cdot(\sec x)^2\,dx$

$\sec^2 x$

$\left(\frac{1}{7}\right)(\tan x)^7+C$

$\frac{\tan^7 x}{7}+C$

D ____ 9. $\int_0^1 \tan x\,dx=$

(A) 0 (B) $\frac{\tan^2 1}{2}$ (C) $\ln(\cos(1))$ (D) $\ln(\sec(1))$ (E) $\ln(\sec(1))-1$

$\int_0^1 \tan x\,dx$

$-\ln|\cos x|\Big|_0^1$

$-[\ln|\cos 1|-\ln|\cos 0|]$

$-[\ln(\cos 1)-\ln(1)]$ * cos 1 > 0 so drop abs. value

$-[\ln(\cos 1)-0]$

$-\ln(\cos 1)$

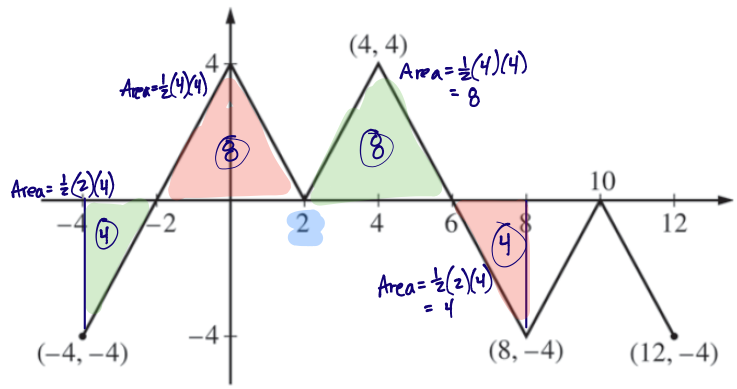
$\ln(\cos 1)^{-1}$

$\ln\left(\frac{1}{\cos 1}\right)$

$\ln(\sec 1)$

PART II: Free Response. SHOW ALL WORK AND/OR INTEGRAL SET-UPS in the space provided. Focus on notation, notation, notation. Communicate, clearly, your result.

10.



Graph of f

The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function $F(x) = \int_2^x f(t)\,dt$.

(a) Evaluate $F(2)$, $F(8)$, and $F(-4)$. Show the work that leads to your answer.

$F(2) = \int_2^2 f(t)\,dt$

$\boxed{F(2) = 0}$ √1

$F(8) = \int_2^8 f(t)\,dt$

$\boxed{F(8) = 8 - 4 = 4}$ √2

$F(-4) = \int_2^{-4} f(t)\,dt$

$\boxed{F(-4) = -8 + 4 = -4}$ √3

(b) Find $F'(x)$ and $F'(0)$.

$F(x) = \int_2^x f(t)\,dt$

$F'(x) = f(x)(1) - f(2)(0)$

$\boxed{F'(x) = f(x)}$ √4

$\boxed{F'(0) = f(0) = 4}$ √5

(c) Find the x-coordinate of any local minimum of $F(x)$. Justify.

$\boxed{F'(x) = f(x) = 0}$ √6

$x = -2, 2, 6, 10$

F(x) has a local minimum at x = −2, since F'(x) = f(x) changes from neg to pos at x = −2 √7

(d) Does the graph of $F(x)$ have and inflection point at $x = 4$? Justify.

$F'(x) = f(x)$

$F''(x) = f'(x) =$ slopes of $f(x)$

$F''(4) = f'(4) =$ DNE (cusp)
So, $x = 4$ is a possible inflection value √8

Since the slopes of f(x) change from pos to neg at x = 4, F(x) has an inflection point at x = 4. √9