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AP Calculus BC TEST: 4.1—4.4 **Calculator Permitted**

PART I: Multiple Choice. SHOW ALL WORK AND/OR INTEGRAL SET-UPS. NO WORK, NO CREDIT. Put the Capital Letter of the correct answer choice in the space to the left of each problem number.

_____ 1. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1)=0$, then $F(9)=$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

_____ 2. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table below. Using the subintervals indicated by the data, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

x	2	5	7	8
$f(x)$	10	30	40	20

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

_____ 3. $\int \frac{\sin\sqrt{9x}}{\sqrt{x}}dx=$

(A) $\cos\sqrt{9x}+C$ (B) $-\cos\sqrt{9x}+C$ (C) $\frac{2}{3}\cos\sqrt{9x}+C$ (D) $-\frac{2}{3}\cos\sqrt{9x}+C$ (E) $\frac{2}{3}\sin^2\sqrt{9x}+C$

_____ 4. $\int \frac{1}{e^x \cot e^{-x}}\,dx =$

(A) $-\ln\left|\cot\left(e^{-x}\right)\right|+C$ (B) $-\ln\left|\sin\left(e^{-x}\right)\right|+C$ (C) $\ln\left|\sin\left(e^{-x}\right)\right|+C$

(D) $-\ln\left|\cos\left(e^{-x}\right)\right|+C$ (E) $\ln\left|\cos\left(e^{-x}\right)\right|+C$

_____ 5. Using the substitution $u=2x+1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

(A) $\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2 \sqrt{u}\,du$ (C) $\frac{1}{2}\int_1^5 \sqrt{u}\,du$ (D) $\int_0^2 \sqrt{u}\,du$ (E) $\int_1^5 \sqrt{u}\,du$

_____ 6. $\frac{d}{dx}\left(\int_0^{x^2} \sin\left(t^3\right)dt\right) =$

(A) $-\cos\left(x^6\right)$ (B) $\sin\left(x^3\right)$ (C) $\sin\left(x^6\right)$ (D) $2x\sin\left(x^3\right)$ (E) $2x\sin\left(x^6\right)$

_____ 7. The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t)=e^t+te^t$. What is the average velocity of the particle from time $t=0$ to $t=3$?

(A) 20.086 ft/sec (B) 26.447 ft/sec (C) 32.809 ft/sec (D) 40.671 ft/sec (E) 79.342 ft/sec

_____ 8. An antiderivative for $\dfrac{1}{x^2-2x+2}$ is

(A) $-\left(x^2-2x+2\right)^{-2}$ (B) $\ln\left|x^2-2x+2\right|$ (C) $\ln\left|\dfrac{x-2}{x+1}\right|$ (D) $\operatorname{arcsec}(x-1)$ (E) $\arctan(x-1)$

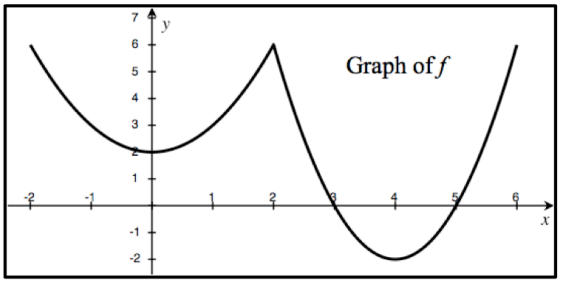
_____ 9. $\displaystyle\int x^2\left(1+\cos\left(1-2x^3\right)\right)dx=$

(A) $\dfrac{x^3}{3}+\dfrac{\sin\left(1-2x^3\right)}{6}+C$ (B) $\dfrac{x^3}{3}-\dfrac{\sin\left(1-2x^3\right)}{6}+C$ (C) $x+\dfrac{\sin\left(1-2x^3\right)}{6}+C$

(D) $x-\dfrac{\sin\left(1-2x^3\right)}{6}+C$ (E) $\dfrac{x}{6}\left(1-\sin\left(1-2x^3\right)\right)+C$

_____ 10. If $f''(x) = -2x$, $f'(3) = 1$, and $f(3) = 20$, find $f(1) - f(-1)$.

(A) $\frac{58}{3}$ (B) $\frac{52}{3}$ (C) 20 (D) $\frac{2}{3}$ (E) 0



_____ 11. Let f be the function in the graph above on the interval $[-2, 6]$. If $F(x) = \int_0^x f(t)\,dt$, on what interval(s) is F decreasing?

(A) None (B) $(3, 4)$ only (C) $(3, 5)$ only (D) $(-2, 0) \cup (2, 4)$ (E) $(-2, 0) \cup (2, 3)$

_____12. Let $f(x) = x^2$ and $g(x) = \sin x$. If $h(x) = \int_{-1}^{g(x)} f(t)\,dt$, find the value of $h'\left(\frac{\pi}{6}\right)$.

(A) $\sqrt{3}$ (B) $\left(2 + \frac{\pi}{3}\right)\left(\frac{\sqrt{3}}{2}\right)$ (C) $\frac{\pi\sqrt{3}}{6}$ (D) $\frac{\sqrt{3}}{8}$ (E) $\frac{1}{4}$

_____13. Let $f(x)=\frac{1}{x}$. For how many value(s) of $b>1$ does the average rate or change of f on the interval $1\le x\le b$ equal the average value of f on the interval $1\le x\le b$?

(A) 0 (B) 1 (C) 2 (D) infinitely many (E) it depends on b

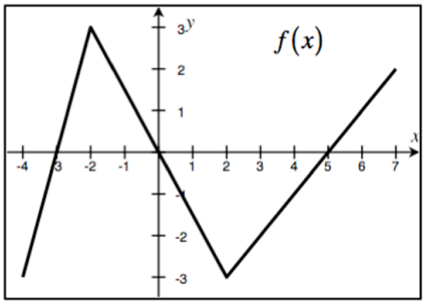
_____ 14. If $\int_{-4}^{2} f(x)\,dx=6$, find $\int_{-2}^{4}\left[f(x-2)+2\right]dx$.

(A) 24 (B) 18 (C) 12 (D) 8 (E) 6

_____ 15. $\int \frac{(x-1)^2}{6\left(x^3-3x^2+3x+12\right)^2}dx=$

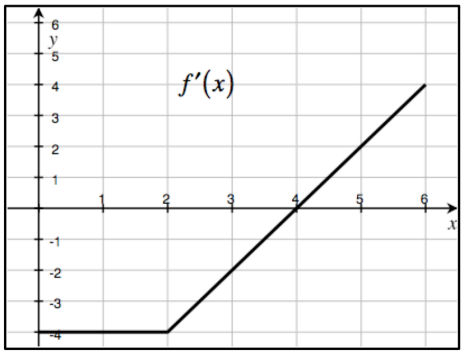
(A) $\frac{-1}{18\left(x^3-3x^2+3x+12\right)}+C$ (B) $\frac{-1}{2\left(x^3-3x^2+3x+12\right)}+C$ (C) $\frac{-2}{x^3-3x^2+3x+12}+C$

(D) $\frac{-1}{54\left(x^3-3x^2+3x+12\right)^3}+C$ (E) $\frac{-1}{6\left(x^3-3x^2+3x+12\right)^3}+C$



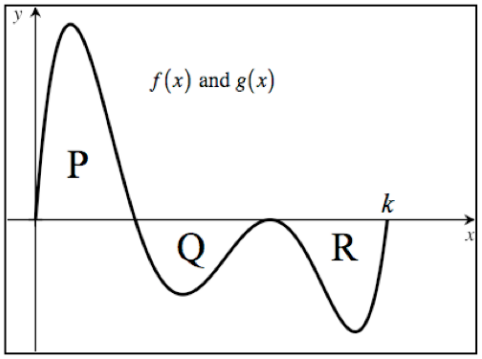
_____ 16. The graph of the piecewise linear function f is shown in the figure above. If $g(x)=\int_{-1}^{x} f(t)\,dt$, which of the following has the smallest value?

(A) $g(-4)$ (B) $g(-3)$ (C) $g(7)$ (D) $g(5)$ (E) $g(0)$



_____ 17. The graph of $f'(x)$, the derivative of f, is shown in the figure above. If $f(0)=3$, then $f(5)=?$

(A) -8 (B) -11 (C) 5 (D) 2 (D) -14



_____ 18. For $x \geq 0$, let $f(x) = g(x)$. The graph above, then, is the graph of both $f(x)$ and $g(x)$ for $x \geq 0$. Let P, Q, and R be positive numbers that represent the areas of the indicated regions bounded by the curve and the x-axis. If $f(x)$ is an even function and $g(x)$ is an odd function, find the value of $\int_0^{-k} [f(x) - g(x)]\,dx$.

(A) $-2P$ (B) $2P$ (C) $2P - 2R - 2Q$ (D) $2R + 2Q - 2P$ (E) 0

x	-2	$-2 < x < 0$	0	$0 < x < 1$	1	$1 < x < 3$	3	$3 < x < 4$	4	$4 < x < 5$	5	$5 < x < 6$	6
$f(x)$	0	+	3	+	0	–	–4	–	–2	–	0	–	–1
$f'(x)$	2	+	DNE	–	–2	–	0	+	1	+	0	–	–3
$f''(x)$	4	+	DNE	+	2	+	1	+	0	–	–1	–	–4

_____ 19. Let $f(x)$ be a function that is continuous on the interval $[-2, 6]$ and twice-differentiable for all $x \neq 0$. The functions f, f', and f'' have values and signs given in the table above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, find the value(s) of x where $g(x)$ has an inflection point.

(A) $x = 1$ only (B) $x = 1$ and $x = 5$ (C) $x = 4$ only (D) $x = 0$, $x = 3$, and $x = 5$ (E) $x = 3$ and $x = 5$

_____ 20. $\int \dfrac{x^3}{\sqrt{(x+1)(x-1)(x^2+1)}}\,dx =$

(A) $\dfrac{-\sqrt{x^4-1}}{4} + C$ (B) $\dfrac{-\sqrt{x^4-1}}{2} + C$ (C) $\dfrac{\sqrt{x^4-1}}{4} + C$ (D) $\dfrac{\sqrt{x^4-1}}{8} + C$ (E) $\dfrac{\sqrt{x^4-1}}{2} + C$

PART II: Free Response

21. Evaluate the following. Show any rewriting and intermediate steps. Don't forget your +C!!

(a) $\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx =$

(b) $\int \frac{e^x}{3+e^x}\,dx =$

(c) $\int \frac{x+1}{\left(x^2+2x+2\right)^3}\,dx =$

(d) $\int \frac{2x^2}{\sqrt{x+1}}\,dx =$

(e) $\int \frac{x^2}{1+x^2}dx =$

(f) $\int \frac{4}{5x\sqrt{x^2-3}}dx =$

(g) $\int \frac{t^3}{\sqrt{1-t^8}}\,dt =$

(h) $\int\left(\frac{4x+3\sqrt[3]{x}-x^2}{2x}\right)dx =$

(i) $\int\left[\cos^2 x+\tan^2 x\right]dx =$