Name KEY ______________________ Date __________ Favorite Bulb Wattage _____

AP Calculus BC TEST: 4.1—4.4 **Calculator Permitted**

PART I: Multiple Choice. SHOW ALL WORK AND/OR INTEGRAL SET-UPS. NO WORK, NO CREDIT. Put the Capital Letter of the correct answer choice in the space to the left of each problem number.

C 1. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$ $= F'(x)$. If $F(1)=0$, then $F(9)=$

(A) 0.048 (B) 0.144 [✓ (C) 5.827] (D) 23.308 (E) 1,640.250

$$F(9) = F(1) + \int_1^9 F'(x)\,dx$$

$$= 0 + \int_1^9 \left(\frac{(\ln x)^3}{x}\right)dx$$

$$= 5.827$$

C 2. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table below. Using the subintervals indicated by the data, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

3 2 1

x	2	5	7	8
$f(x)$	10	30	40	20

(A) 110 (B) 130 [✓ (C) 160] (D) 190 (E) 210

$$\int_2^8 f(x)\,dx \approx \frac{1}{2}\left[3(10+30)+2(30+40)+1(40+20)\right]$$

$$= \frac{1}{2}\left[3(40)+2(70)+60\right]$$

$$= 160$$

D 3. $\int \frac{\sin\sqrt{9x}}{\sqrt{x}}dx =$

(A) $\cos\sqrt{9x}+C$ (B) $-\cos\sqrt{9x}+C$ (C) $\frac{2}{3}\cos\sqrt{9x}+C$ [✓ (D) $-\frac{2}{3}\cos\sqrt{9x}+C$] (E) $\frac{2}{3}\sin^2\sqrt{9x}+C$

$$\int \left(\sin \underline{3x^{1/2}}\right)\left(x^{-1/2}\right)$$

$$\rightarrow \frac{3}{2}x^{-1/2} \text{ (off by } \tfrac{3}{2})$$

$$\frac{2}{3}\left(-\cos\left(3x^{1/2}\right)\right)+C$$

correction

$$\boxed{-\frac{2}{3}\cos\sqrt{9x}+C}$$

E 4. $\int \frac{1}{e^{x}\cot e^{-x}}\,dx =$

(A) $-\ln\left|\cot\left(e^{-x}\right)\right|+C$ (B) $-\ln\left|\sin\left(e^{-x}\right)\right|+C$ (C) $\ln\left|\sin\left(e^{-x}\right)\right|+C$

(D) $-\ln\left|\cos\left(e^{-x}\right)\right|+C$ [✓ (E) $\ln\left|\cos\left(e^{-x}\right)\right|+C$]

$\int e^{-x}\cdot\tan(e^{-x})\,dx$

$-e^{-x}$ (off by a neg)

$-(-\ln|\cos(e^{-x})|)+C$

$\ln|\cos(e^{-x})|+C$

C 5. Using the substitution $u=2x+1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

(A) $\frac{1}{2}\int_{-1/2}^{1/2}\sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2\sqrt{u}\,du$ [✓ (C) $\frac{1}{2}\int_1^5\sqrt{u}\,du$] (D) $\int_0^2\sqrt{u}\,du$ (E) $\int_1^5\sqrt{u}\,du$

$u=2x+1$

$du=2dx$

$dx=\frac{1}{2}du$

$\frac{1}{2}(u-1)=x$

when $x=0, u=1$

when $x=2, u=5$

$\int_1^5 \sqrt{u}\cdot\frac{1}{2}du$

$\boxed{\frac{1}{2}\int_1^5\sqrt{u}\,du}$

E 6. $\frac{d}{dx}\left(\int_0^{x^2}\sin\left(t^3\right)dt\right) = \sin((x^2)^3)\cdot(2x) - 0 = 2x\sin(x^6)$

(A) $-\cos\left(x^6\right)$ (B) $\sin\left(x^3\right)$ (C) $\sin\left(x^6\right)$ (D) $2x\sin\left(x^3\right)$ [✓ (E) $2x\sin\left(x^6\right)$]

A 7. The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to $t = 3$?

(A) 20.086 ft/sec (B) 26.447 ft/sec (C) 32.809 ft/sec (D) 40.671 ft/sec (E) 79.342 ft/sec

$$\text{Avg} = \frac{\int_0^3 v(t)\,dt}{3-0}$$

$$= \frac{1}{3}\int_0^3 (e^t + te^t)\,dt$$

$$= 20.0855\ldots$$

E 8. An antiderivative for $\dfrac{1}{x^2 - 2x + 2}$ is

(A) $-\left(x^2 - 2x + 2\right)^{-2}$ (B) $\ln\left|x^2 - 2x + 2\right|$ (C) $\ln\left|\dfrac{x-2}{x+1}\right|$ (D) $\operatorname{arcsec}(x-1)$ (E) $\arctan(x-1)$

$$\int \frac{1}{(x^2 - 2x + \underline{1}) \underline{-1} + 2}\,dx$$

$$\int \frac{1}{\underset{u=x-1}{(x-1)^2} + \underset{a=1}{1}}\,dx$$

$$\arctan(x-1) + C$$

B 9. $\int x^2\left(1 + \cos\left(1 - 2x^3\right)\right)dx =$

(A) $\dfrac{x^3}{3} + \dfrac{\sin\left(1-2x^3\right)}{6} + C$ (B) $\dfrac{x^3}{3} - \dfrac{\sin\left(1-2x^3\right)}{6} + C$ (C) $x + \dfrac{\sin\left(1-2x^3\right)}{6} + C$

(D) $x - \dfrac{\sin\left(1-2x^3\right)}{6} + C$ (E) $\dfrac{x}{6}\left(1 - \sin\left(1-2x^3\right)\right) + C$

$$\int\left(x^2 + x^2\cos(\underline{1-2x^3})\right)dx$$

$$-6x^2 \text{ (off by } -6)$$

$$\downarrow$$

$$\frac{1}{3}x^3 - \frac{1}{6}\sin(1-2x^3) + C$$

A 10. If $f''(x)=-2x$, $f'(3)=1$, and $f(3)=20$, find $f(1)-f(-1)$.

(A) $\frac{58}{3}$ (B) $\frac{52}{3}$ (C) 20 (D) $\frac{2}{3}$ (E) 0

$f'(x)=-x^2+C$

for $f'(3)=1$: $1=-(3^2)+C$

$1=-9+C$

$C=10$

So, $f'(x)=-x^2+10$

$f(x)=-\frac{1}{3}x^3+10x+C$

for $f(3)=20$: $20=(-\frac{1}{3})(27)+30+C$

$20=-9+30+C$

$20=21+C$

$C=-1$

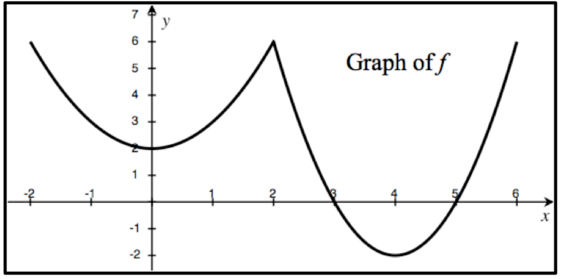
So, $f(x)=-\frac{1}{3}x^3+10x-1$

$f(1)=-\frac{1}{3}+10-1=-\frac{1}{3}+9=\frac{26}{3}$

$f(-1)=\frac{1}{3}-10-1=\frac{1}{3}-11=-\frac{32}{3}$

$\frac{26}{3}-(-\frac{32}{3})$

$\frac{58}{3}$



C 11. Let f be the function in the graph above on the interval $[-2,6]$. If $F(x)=\int_0^x f(t)\,dt$, on what interval(s) is F decreasing?

(A) None (B) $(3,4)$ only (C) $(3,5)$ only (D) $(-2,0)\cup(2,4)$ (E) $(-2,0)\cup(2,3)$

F is decreasing when $F'=f$ is negative

$f<0$ for $3<x<5$

D 12. Let $f(x)=x^2$ and $g(x)=\sin x$. If $h(x)=\int_{-1}^{g(x)} f(t)\,dt$, find the value of $h'\left(\frac{\pi}{6}\right)$.

$f(\frac{1}{2})=(\frac{1}{2})^2=\frac{1}{4}$

(A) $\sqrt{3}$ (B) $\left(2+\frac{\pi}{3}\right)\left(\frac{\sqrt{3}}{2}\right)$ (C) $\frac{\pi\sqrt{3}}{6}$ (D) $\frac{\sqrt{3}}{8}$ (E) $\frac{1}{4}$

$g(\frac{\pi}{6})=\sin\frac{\pi}{6}$

$=\frac{1}{2}$

$g'(x)=\cos x$

$g'(\frac{\pi}{6})=\cos\frac{\pi}{6}$

$=\frac{\sqrt{3}}{2}$

$h'(x)=f(g(x))\cdot g'(x)$

$h'(\frac{\pi}{6})=f(g(\frac{\pi}{6}))\cdot g'(\frac{\pi}{6})$

$=f(\frac{1}{2})\cdot(\frac{\sqrt{3}}{2})$

$=(\frac{1}{4})(\frac{\sqrt{3}}{2})$

$=\frac{\sqrt{3}}{8}$

B 13. Let $f(x)=\frac{1}{x}$. For how many value(s) of $b>1$ does the average rate or change of f on the interval $1\le x\le b$ equal the average value of f on the interval $1\le x\le b$?

(A) 0 [✓ (B) 1] (C) 2 (D) infinitely many (E) it depends on b

Avg value $= \dfrac{\int_1^b \frac{1}{x}\,dx}{b-1} = \frac{1}{b}\ln|x|\Big|_1^b = \frac{1}{b}[\ln b-\ln 1] = \frac{1}{b}\ln b$

Avg-R-o-c $= \dfrac{\frac{1}{b}-\frac{1}{1}}{b-1} = \dfrac{\frac{1}{b}-1}{b-1}\left(\frac{b}{b}\right) = \dfrac{(1-b)}{-(1-b)b} = \frac{-1}{b}$

so $\frac{1}{b}\ln b = \frac{-1}{b}$, $b>1$

$\ln b=-1$

$b=e^{-1}$

B 14. If $\int_{-4}^{2} f(x)\,dx=6$, find $\int_{-2}^{4}[f(x-2)+2]\,dx$.

(A) 24 [✓ (B) 18] (C) 12 (D) 8 (E) 6

$\int_{-2}^{4} f(x-2)\,dx+\int_{-2}^{4}2\,dx$

Let $u=x-2$

$du=dx$

$x=u+2$

when $x=-2, u=-4$

when $x=4, u=2$

$\int_{-4}^{2} f(u)\,du+2x\Big|_{-2}^{4}$

$6+2[4-(-2)]$

$6+2[6]$

$6+12$

18

A 15. $\displaystyle\int\frac{(x-1)^2}{6\left(x^3-3x^2+3x+12\right)^2}\,dx=$

[✓ (A) $\dfrac{-1}{18\left(x^3-3x^2+3x+12\right)}+C$] (B) $\dfrac{-1}{2\left(x^3-3x^2+3x+12\right)}+C$ (C) $\dfrac{-2}{x^3-3x^2+3x+12}+C$

(D) $\dfrac{-1}{54\left(x^3-3x^2+3x+12\right)^3}+C$ (E) $\dfrac{-1}{6\left(x^3-3x^2+3x+12\right)^3}+C$

$\frac{1}{6}\int(x^2-2x+1)(x^3-3x^2+3x+12)^{-2}\,dx = \left(\frac{1}{6}\right)\left(\frac{1}{3}\right)\left(-1\right)(x^3-3x^2+3x+12)^{-1}+C$

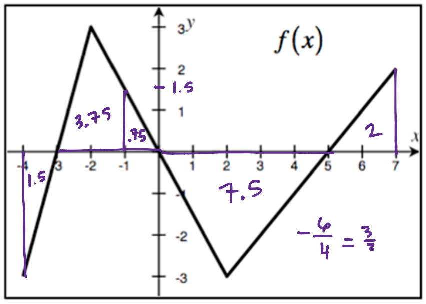
rider correction rule

$3x^2-6x+3$

$=3(x^2-2x+1)$

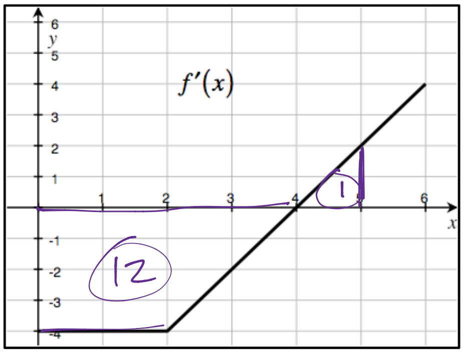
(off by 3)

$= \dfrac{-1}{18(x^3-3x^2+3x+12)}+C$



D 16. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-1}^{x} f(t)\,dt$, which of the following has the smallest value?

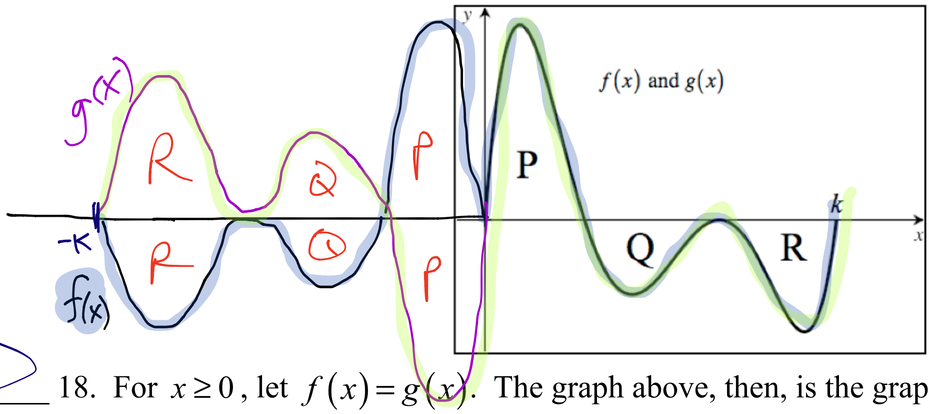
(A) $g(-4)$	(B) $g(-3)$	(C) $g(7)$	[✓ (D) $g(5)$]	(E) $g(0)$
$\int_{-1}^{-4} f(x)\,dx$	$\int_{-1}^{-3} f(x)\,dx$	$\int_{-1}^{7} f(x)\,dx$	$\int_{-1}^{5} f(x)\,dx$	$\int_{-1}^{0} f(x)\,dx$
$-3.75+1.5$	-3.75	$.75-7.5+2$	$.75-7.5$	$.75$
-2.25		-4.75	-6.75	



A 17. The graph of $f'(x)$, the derivative of f, is shown in the figure above. If $f(0) = 3$, then $f(5) = ?$

[✓ (A) -8] (B) -11 (C) 5 (D) 2 (D) -14

$$f(5) = f(0) + \int_0^5 f'(x)\,dx$$
$$= 3 + (-12) + 1$$
$$= 4 - 12$$
$$= -8$$



D

18. For $x \geq 0$, let $f(x) = g(x)$. The graph above, then, is the graph of both $f(x)$ and $g(x)$ for $x \geq 0$. Let P, Q, and R be positive numbers that represent the areas of the indicated regions bounded by the curve and the x-axis. If $f(x)$ is an even function and $g(x)$ is an odd function, find the value of $\int_0^{-k} [f(x) - g(x)]\,dx$.

(A) $-2P$ (B) $2P$ (C) $2P - 2R - 2Q$ (D) $2R + 2Q - 2P$ (E) 0

$\int_0^{-K} f(x)\,dx - \int_0^{-K} g(x)\,dx$

$-\int_{-K}^{0} f(x)\,dx + \int_{-K}^{0} g(x)\,dy$

$\int_{-K}^{0} g(x)\,dx - \int_{-K}^{0} f(x)\,dx$

$(R+Q-P) - (-R-Q+P)$

$R+Q-P+R+Q-P$

$2R+2Q-2P$

x	-2	$-2<x<0$	0	$0<x<1$	1	$1<x<3$	3	$3<x<4$	4	$4<x<5$	5	$5<x<6$	6
$f(x)$	0	+	3	+	0	–	–4	–	–2	–	0	–	–1
$f'(x)$	2	+	DNE	–	–2	–	0	+	1	+	0	–	–3
$f''(x)$	4	+	DNE	+	2	+	1	+	0	–	–1	–	–4

D

19. Let $f(x)$ be a function that is continuous on the interval $[-2,6]$ and twice-differentiable for all $x \neq 0$. The functions f, f', and f'' have values and signs given in the table above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, find the value(s) of x where $g(x)$ has an inflection point.

(A) $x=1$ only (B) $x=1$ and $x=5$ (C) $x=4$ only (D) $x=0$, $x=3$, and $x=5$ (E) $x=3$ and $x=5$

$g'(x) = f(x)$

$g''(x) = f'(x)$

$g'' = f'(x)$ changes signs

E

20. $\int \frac{x^3}{\sqrt{(x+1)(x-1)(x^2+1)}}\,dx =$

(A) $\frac{-\sqrt{x^4-1}}{4}+C$ (B) $\frac{-\sqrt{x^4-1}}{2}+C$ (C) $\frac{\sqrt{x^4-1}}{4}+C$ (D) $\frac{\sqrt{x^4-1}}{8}+C$ (E) $\frac{\sqrt{x^4-1}}{2}+C$

$\int \frac{x^3}{\sqrt{(x^2-1)(x^2+1)}}\,dx$

$\int \frac{x^3}{\sqrt{x^4-1}}\,dx$

$\int x^3 (x^4-1)^{-1/2}\,dx$

$4x^3$ (off by 4)

$\left(\frac{1}{4}\right)(2)(x^4-1)^{1/2}+C$

corr rule

$\frac{\sqrt{x^4-1}}{2}+C$

PART II: Free Response

21. Evaluate the following. Show any rewriting and intermediate steps. Don't forget your +C!!

(a) $\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx =$

$\int \sec^2 x\,(\tan x)^{-1/2}\,dx$

$\boxed{2\sqrt{\tan x} + C}$

(b) $\int \frac{e^x}{3+e^x}\,dx =$

$\boxed{\begin{array}{c} \ln|3+e^x| + C \\ \text{or} \\ \ln(3+e^x) + C \end{array}}$

Since $3+e^x > 0\ \forall x \in \mathbb{R}$

(c) $\int \frac{x+1}{(x^2+2x+2)^3}\,dx =$

$\int (x+1)(x^2+2x+2)^{-3}\,dx$

$2x+2 = 2(x+1)$
(off by 2)

$\left(\frac{1}{2}\right)(-2)(x^2+2x+2)^{-2} + C$

corr rule

$\boxed{\frac{-1}{(x^2+2x+2)^2} + C}$

(d) $\int \frac{2x^2}{\sqrt{x+1}}\,dx =$

$u = x+1$
$du = dx$
$x = u-1$

$2\int \frac{(u-1)^2}{\sqrt{u}}\,du$

$2\int \left(\frac{u^2}{u^{1/2}} - \frac{2u}{u^{1/2}} + \frac{1}{u^{1/2}}\right)du$

$2\int \left(u^{3/2} - 2u^{1/2} + u^{-1/2}\right)du$

$\boxed{\begin{array}{l} 2\left[\frac{2}{5}(x+1)^{5/2} - (2)\left(\frac{2}{3}\right)(x+1)^{3/2} + 2(x+1)^{1/2}\right] + C \\ \text{or } \frac{4}{5}(x+1)^{5/2} - \frac{8}{3}(x+1)^{3/2} + 4(x+1)^{1/2} + C \\ \text{or} \\ \frac{4}{15}(x+1)^{1/2}\left[3(x+1)^2 - 2(x+1) + 15\right] + C \end{array}}$

(e) $\int \frac{x^2}{1+x^2}\,dx =$

$\int \frac{x^2}{x^2+1}\,dx$

$x^2+1 \,\overline{)\, x^2}$ quotient 1
$-x^2 - 1$
-1

$\int \left(1 - \frac{1}{x^2+1}\right)dx$

$\boxed{x - \arctan x + C}$

(f) $\int \frac{4}{5x\sqrt{x^2-3}}\,dx =$

$\frac{4}{5}\int \frac{1}{x\sqrt{x^2-3}}\,dx$

$u = x \quad a = \sqrt{3}$

$\boxed{\begin{array}{c} \left(\frac{4}{5}\right)\left(\frac{1}{\sqrt{3}}\right)\operatorname{arcsec}\left(\frac{x}{\sqrt{3}}\right) + C \\ \frac{4}{5\sqrt{3}}\operatorname{arcsec}\left(\frac{x}{\sqrt{3}}\right) + C \end{array}}$

(g) $\int \frac{t^3}{\sqrt{1-t^8}}\,dt =$

$a = 1 \quad u = t^4 \quad u' = 4t^3$
(off by 4)

$\boxed{\begin{array}{l} \left(\frac{1}{4}\right)\arcsin\left(\frac{t^4}{1}\right) + C \\ \text{corr} \\ \frac{1}{4}\arcsin t^4 + C \end{array}}$

(h) $\int \left(\frac{4x + 3\sqrt[3]{x} - x^2}{2x}\right)dx =$

$\int \left(\frac{4x}{2x} + \frac{3x^{1/3}}{2x} - \frac{x^2}{2x}\right)dx$

$\int \left(2 + \frac{3}{2}x^{-2/3} - \frac{1}{2}x\right)dx$

$\boxed{\begin{array}{l} 2x + \left(\frac{3}{2}\right)(3)x^{1/3} - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)x^2 + C \\ 2x + \frac{9}{2}x^{1/3} - \frac{1}{4}x^2 + C \end{array}}$

(i) $\int \left[\cos^2 x + \tan^2 x\right]dx =$

$\int \left[\frac{1}{2}(1+\cos 2x) + (\sec^2 x - 1)\right]dx$

$\boxed{\begin{array}{l} \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + \tan x - x + C \\ \frac{1}{2}x + \frac{1}{4}\sin 2x + \tan x - x + C \\ \frac{1}{4}\sin 2x + \tan x - \frac{1}{2}x + C \end{array}}$