Name KEY Date______________ Per_____

TEST: AP Calculus: Test—6.1-6.4 NO CALCUALTOR

PART I: Multiple Choice. Put the Capital Letter of the correct answer choice in the space to the left of each problem number.

FREE 1. Let $F(x)=\int_1^x \frac{(\ln x)^3}{x}dx$, what is the value of $F(e)$? 6.3 #7

(A) 0 (B) 1 (C) 2 (D) e (E) DNE

$\frac{1}{4}(\ln x)^4\Big|_1^e = \frac{1}{4}[1] = \frac{1}{4}$

C 2. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table below. Using the subintervals indicated by the data, what is the **trapezoidal** approximation of $\int_2^8 f(x)dx$?

$\frac{1}{2}[3(40)+2(70)+1(60)]$

$\frac{1}{2}[120+140+60]$

$\frac{1}{2}[320] = \boxed{160}$

x	2	5	7	8
$f(x)$	10	30	40	20

(A) 210 (B) 190 (C) 160 (D) 130 (A) 110

A 3. If f is continuous on the interval $[a,b]$, then there exists c such that $a \le c \le b$ and $\int_a^b f(x)dx=$

(A) $f(c)(b-a)$ (B) $f'(c)(b-a)$ (C) $f(b)-f(a)$ (D) $\frac{f(b)-f(a)}{b-a}$ (E) $\frac{f(c)}{b-a}$

6.3 #12

C 4. $\int x^2\cos(x^3)dx=$ $\frac{1}{3}\sin(x^3)+c$

(A) $\sin\left(\frac{x^4}{4}\right)+C$ (B) $-\frac{1}{3}\sin(x^3)+C$ (C) $\frac{1}{3}\sin(x^3)+C$

(D) $-\frac{x^3}{3}\sin(x^3)+C$ (E) $\frac{x^3}{3}\sin(x^3)+C$

$\frac{1}{2}\int_1^5 \sqrt{u}\,du$

B 5. Using the substitution $u = 2x+1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

(A) $\frac{1}{2}\int_0^2 \sqrt{u}\,du$ (B) $\frac{1}{2}\int_1^5 \sqrt{u}\,du$ (C) $\int_0^2 \sqrt{u}\,du$ (D) $\int_1^5 \sqrt{u}\,du$ (E) $\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\,du$

B 6. $\frac{d}{dx}\left(\int_0^{x^2} \sin\left(t^3\right)dt\right) =$ 6.3 # 9, 10, 11

(A) $-\cos\left(x^6\right)$ (B) $2x\sin\left(x^6\right)$ (C) $\sin\left(x^3\right)$ (D) $2x\sin\left(x^3\right)$ (E) $\sin\left(x^6\right)$

$2x\sin(x^6)$

C 7. The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = t^2 + 1$. What is the **average velocity** of the particle from time $t = 0$ to $t = 3$?

(A) 12 ft/sec (B) 3 ft/sec (C) 4 ft/sec (D) $\sqrt{3}$ ft/sec (E) $\frac{10}{3}$ ft/sec

6.3 ex 14 $\frac{\frac{1}{3}t^3 + t \Big|_0^3}{3} = \frac{1}{3}\left[(9+3)\right] = \boxed{4}$

D 8. An antiderivative for $\frac{1}{x^2 - 2x + 2}$ is $\frac{1}{x^2 - 2x + 1 - 1 + 2}$

(A) $-\left(x^2 - 2x + 2\right)^{-2}$ (B) $\ln\left|x^2 - 2x + 2\right|$ (C) $\operatorname{arcsec}(x-1)$ (D) $\arctan(x-1)$ (E) $\ln\left|\frac{x-2}{x+1}\right|$

$\frac{1}{(x-1)^2 + 1}$

A 9. $\int \frac{x}{x^2 - 4}\,dx =$ $\frac{1}{2}\ln\left|x^2 - 4\right| + C$

(A) $\frac{1}{2}\ln\left|x^2 - 4\right| + C$ (B) $\frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$ (C) $2\ln\left|x^2 - 4\right| + C$

(D) $\frac{-1}{4\left(x^2 - 4\right)^2} + C$ (E) $\frac{1}{2\left(x^2 - 4\right)} + C$

(1) FREE (5) B
(2) C (6) B
(3) A (7) C
(4) C (8) D
(9) A

PART II: Free Response

10. Evaluate 6 of the 8 of the following. Don't forget your +C!!

(a) $\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx$

$\int \sec^2 x \cdot (\tan x)^{-1/2}\,dx$

$\boxed{2(\tan x)^{1/2} + C \quad \text{or} \quad 2\sqrt{\tan x} + C}$

(b) $\int \frac{e^x}{3+e^x}\,dx$

$\boxed{\ln|3+e^x| + C \quad \text{or} \quad \ln(3+e^x) + C}$

(c) $\int \frac{x+1}{\left(x^2+2x+2\right)^3}\,dx$

$\int (x+1)(x^2+2x+2)^{-3}\,dx$

$\boxed{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)(x^2+2x+2)^{-2} + C \quad \text{or} \quad -\frac{1}{4(x^2+2x+2)^2} + C}$

(d) $\int \frac{x}{\sqrt{x+1}}\,dx$

$u = x+1$
$du = dx$
$x = u-1$

$\int (u-1)u^{-1/2}\,du$

$\int \left(u^{1/2} - u^{-1/2}\right)du$

$\boxed{\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + C}$

(e) $\int \frac{x^2}{1+x^2}dx$

$$x^2+1 \overline{\big) x^2} \quad \begin{array}{r} 1 \\ -x^2 - 1 \\ \hline -1 \end{array}$$

$$\int \left(1 - \frac{1}{x^2+1}\right)dx$$

$$\boxed{x - \arctan x + C}$$

(f) $\int \frac{4}{5x\sqrt{x^2-3}}dx$

$$\frac{4}{5}\int \frac{1}{x\sqrt{x^2-3}}$$

$a = \sqrt{3}$

$$\boxed{\left(\frac{4}{5}\right)\left(\frac{1}{\sqrt{3}}\right)\operatorname{arcsec}\frac{|x|}{\sqrt{3}} + C}$$

(g) $\int \frac{t^3}{\sqrt{1-t^8}}dt$

$a = 1 \quad u = t^4$

$$\boxed{\left(\frac{1}{4}\right)\arcsin(t^4) + C}$$

(h) $\int \left(\frac{4x + 3\sqrt[3]{x} - x^2}{2x}\right)dx$

$$\int \left(2 + \frac{3}{2}x^{-2/3} - \frac{1}{2}x\right)dx$$

$$2x + \left(\frac{3}{2}\right)(3)x^{1/3} - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)x^2 + C$$

or

$$2x + \frac{9}{2}x^{1/3} - \frac{1}{4}x^2 + C$$