Name_____ Date_____ Period_____

AP Calculus Test 4.1-4.3, No calculator

Multiple Choice

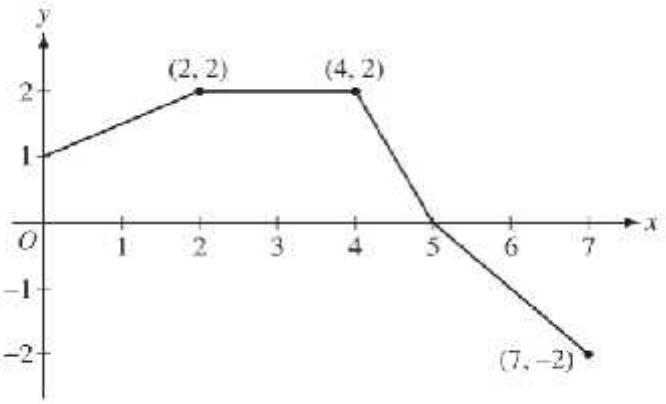
_____ 1. $\int \sec x \tan x dx =$ (A) $\sec x + C$ (B) $\tan x + C$ (C) $\frac{\sec^2 x}{2} + C$ (D) $\frac{\tan^2 x}{2} + C$ (E) $\frac{\sec^2 x \tan^2 x}{2} + C$

_____ 2. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3 \end{cases}$. What is the value of $\int_1^5 f(x)dx$?

(A) 2 (B) 6 (C) 8 (D) 10 (E) 12

____ 3. The graph of a function f is shown at right. What is the value of $\int_0^7 f(x)dx$?

(A) 6 (B) 8 (C) 10 (D) 14 (E) 18



Graph of f

x	0	2	4	6
$f(x)$	4	k	8	12

_____ 4. The function f is continuous on the closed interval $[0,6]$ and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)dx$ found with 3 subintervals of equal length is 52. What is the value of k?

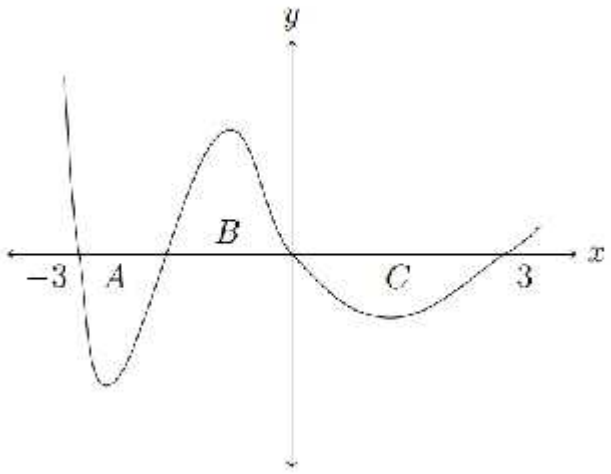
(A) 2 (B) 6 (C) 7 (D) 10 (E) 14

_____ 5. $\int \left(x^3 + 1\right)^2 dx =$ (A) $\frac{1}{7}x^7 + x + C$ (B) $\frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$ (C) $6x^2\left(x^3 + 1\right) + C$

(D) $\frac{1}{3}\left(x^3 + 1\right)^3 + C$ (E) $\frac{\left(x^3 + 1\right)^3}{9x^2} + C$

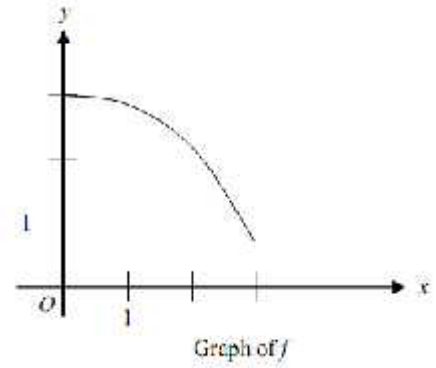
_____ 6. $\int_1^4 |x-3|dx =$ (A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$ (E) 5

_____ 7. The regions *A*, *B*, and *C* in the figure at right are bounded by the graph of the function f and the *x*-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3} (f(x)+1)dx$?



(A) −2 (B) −1 (C) 2 (D) 4 (E) 7

_____ 8. The graph of the function *f* is shown below for $0 \le x \le 3$. Of the following, which has the least value?



Graph of *f*

(A) $\int_1^3 f(x)dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length

(C) Right Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length

(E) Trapezoidal sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length

_____ 9. If $\int_{-5}^{2} f(x)dx = -17$ and $\int_{5}^{2} f(x)dx = -4$, what is the value of $\int_{-5}^{5} f(x)dx$?

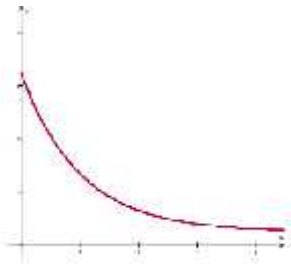
(A) −21 (B) −13 (C) 0 (D) 13 (E) 21

_____ 10. Let f and g be continuous functions for $a \le x \le b$. If $a < c < b$, $\int_a^b f(x)\,dx = P$, $\int_c^b f(x)dx = Q$, $\int_a^b g(x)\,dx = R$, and $\int_c^b g(x)\,dx = S$, then $\int_a^c \left(f(x) - g(x) \right) dx =$

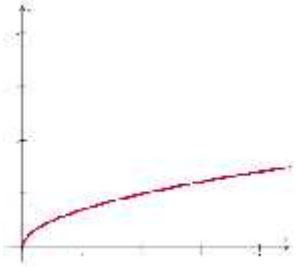
(A) $P - Q + R - S$ (B) $P - Q - R + S$ (C) $P - Q - R - S$ (D) $P + Q - R - S$ (E) $P + Q - R + S$

_____ 11. If a trapezoidal sum over-approximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

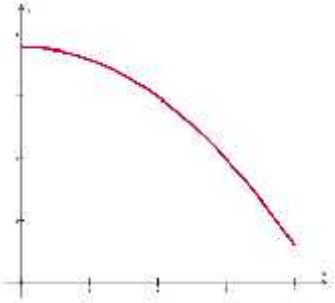
(A)



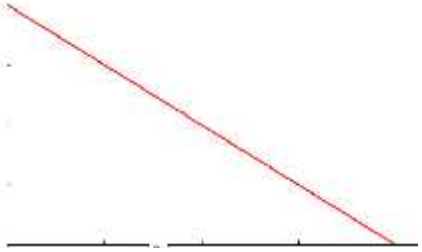
(B)



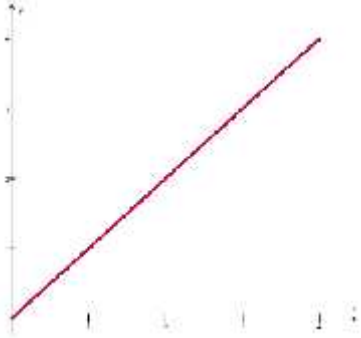
(C)



(D)



(E)



_____ 12. The function f is continuous on the closed interval $[2,13]$ and has values as shown in the table below. Using the intervals $[2,3]$, $[3,5]$, $[5,8]$, and $[8,13]$, what is the approximation of $\int_2^{13} f(x)\,dx$ obtained from a left Riemann sum?

x	2	3	5	8	13
$f(x)$	6	–2	–1	3	9

(A) 6 (B) 14 (C) 28 (D) 32 (E) 50

_____ 13. If $f(x)=g(x)+7$ for $3\le x\le 5$, then $\int_3^5 [f(x)+g(x)]dx =$

(A) $2\int_3^5 g(x)dx+7$ (B) $2\int_3^5 g(x)dx+14$ (C) $2\int_3^5 g(x)dx+28$ (D) $\int_3^5 g(x)dx+7$ (E) $\int_3^5 g(x)dx+14$

_____ 14. The function *f* is continuous on the closed interval $[2,14]$ and has values as show in the table below. Using three subintervals indicated by the data, what is the approximation of $\int_2^{14} f(x)dx$ found by using a right Riemann sum?

x	2	5	10	14
$f(x)$	12	28	34	30

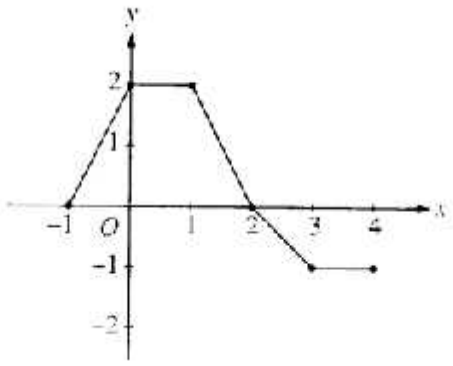
(A) 296 (B) 312 (C) 343 (D) 374 (E) 390

_____ 15. The most general antiderivative of $f(x)=(\sec x)\left(\frac{\cot x}{\sin x}\right)$ is

(A) $\sec x\tan x+C$ (B) $-\csc x\cot x+C$ (C) $-\cot x+C$ (D) $\cos x+C$

_____ 16. If $\int_{-1}^{3} f(x)dx=2$ and $\int_2^3 f(x)dx=-1$, find $\int_{-1}^{2}[2f(x)]dx$

(A) 2 (B) −3 (C) 3 (D) −6 (E) 6



_____ 17. The graph of a piecewise-linear function *f*, for $-1\le x\le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)dx$? (A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

_____ 18. If f is continuous for all x, which of the following integrals necessarily have the same value?

I. $\int_a^b f(x)dx$ II. $\int_0^{b-a} f(x+a)dx$ III. $\int_{a+c}^{b+c} f(x+c)dx$

(A) I and II only (B) I and III only (C) II and III only (D) I, II, and III (E) None

Short Answer: Evaluate the following indefinite integrals. Remember, rewriting is the key, and don't forget your $+C$.
Evaluate 4 of 6 of the following integrals (or get them all correct for amazing bonus points).

12. $\int e \csc x \tan^2 x dx$

13. $\int \frac{2}{5 \cdot 7^{-x}} dx$

14. $\int \left(\frac{4x + 3\sqrt[3]{x} - x^2}{2x} \right) dx$

15. $\int 2\sqrt{x}(3x-2)^2 dx$

16. $\int \left(\frac{4}{\text{®}\, x} - \frac{2}{\sin^2 x} \right) dx$

17. $\int \left(\frac{e^{-x} - 1}{e^{-x}} \right) dx$